Module 1

Introduction to Limit State Design & Serviceability

Contents

Introduction

A structure refers to a system of connected parts used to support forces (loads). Buildings, bridges and towers are examples for structures in civil engineering. In buildings, structure consists of walls floors, roofs and foundation. In bridges, the structure consists of deck, supporting systems and foundations. In towers the structure consists of vertical, horizontal and diagonal members along with foundation.

A structure can be broadly classified as (i) sub structure and (ii) super structure. The portion of building below ground level is known as sub-structure and portion above the ground is called as super structure. Foundation is sub structure and plinth, walls, columns, floor slabs with or without beams, stairs, roof slabs with or without beams etc are super-structure.

Many naturally occurring substances, such as clay, sand, wood, rocks natural fibers are used to construct buildings. Apart from this many manmade products are in use for building construction. Bricks, tiles, cement concrete, concrete blocks, plastic, steel & glass etc are manmade building materials.

Objectives

1. To understand various design philosophies.
2. To understand the necessity of reinforcement in RC structure.
3. To understand the stress block parameter of RC beam section.
4. To understand the necessity of partial safety in design of RC member.

Advantages Disadvantages of RC members

Advantages

- It has high tensile and compressive strength
- It is more durable and may long up to 100 years
- It imparts ductility
- Raw materials used for construction of RC buildings are easily available and can be transported.
- Overall cost for constructing a building using RC proves to be economical compared to steel and pre-stressed structures.
- RC components can be moulded to any desired shape , if formwork is designed properly.
- If RC structures are properly designed then it can resist the earthquake forces.

Disadvantage

- Tensile strength of RC member is about $1/10^{th}$ of its compressive strength

Materials required for RC member

a. Concrete

Concrete is a product obtained artificially by hardening of the mixture of cement, sand, gravel and water in predetermined proportions. Depending on the quality and proportions of the ingredients used in the mix the properties of concrete vary almost as widely as different kinds of stones. Concrete has enough strength in compression, but has little strength in tension. Due to this, concrete is weak in bending, shear and torsion. Hence the use of plain concrete is limited applications where great compressive strength and weight are the principal requirements and where tensile stresses are either totally absent or are extremely low.

Properties of Concrete

1. **Grade of concrete**

Mild	M20
Moderate	M25
Severe	M30
Very Severe	M35
Extreme	M40

2. **Tensile strength**

   $F_{cr} = 0.7 * \sqrt{f_{ck}}$

3. **Modulus of elasticity**

   $E_c = 5000 * \sqrt{f_{ck}}$

4. **Shrinkage of concrete:** Depends on
   - Constituents of concrete
   - Size of the member
   - Environmental conditions
5. **Creep of concrete:** Depends on
   - Strength of the concrete
   - Stress in concrete
   - Duration of loading
6. **Durability:** Mainly depends on
   - Type of Environment
   - Cement content
   - Water cement ratio
   - Workmanship
   - Cover to the reinforcement

7. Cover to the reinforcement

Nominal cover is essential

- Resist corrosion
- Bonding between steel and concrete

b) Reinforcements

- Bamboo, natural fibers (jute, coir etc) and steel are some of the types of reinforcements

Roles of reinforcement in RCC

- To resist Bending moment in case of flexural members
- To reduce the shrinkage of concrete
- To improve the load carrying capacity of the compression member
- To resist the effect of secondary stresses like temperature etc.
- To prevent the development of wider cracks formed due to tensile stress

Advantages of Steel Reinforcement

- It has high tensile and compressive stress
- It is ductile in nature
- It has longer life
- It allows easy fabrication (easy to cut, bend or weld)
- It is easily available
- It has low co-efficient of thermal expansion same as that of concrete

Disadvantages of Steel Reinforcement

- More prone to corrosion
- Loses its strength when exposed to high temperature

Classification of Steel bars

1. Mild Steel plain bars

- Cold worked steel bars
- Hot rolled mild steel bars

Eg: Fe250

2. High Yield Strength Deformed (HYSD) Bars

Eg: Fe415 & Fe500

3. Steel wire Fabric

4. Structural Steel

5. CRS and TMT

Introduction to RCC Design

Objective:-

1. Structure should perform satisfactorily during its life span
2. Structure should take up the forces which are likely and deform within the limit
3. The structure should resist misuse or fire.

Design of RC member involves

1. Deciding the size or dimension of the structural element and amount of reinforcement required.
2. To check whether the adopted size perform satisfactorily during its life span.

Methods of Design or Design philosophy

1. Working stress method
2. Ultimate or load factor method
3. Limit state method

Working Stress Method – Based on Elastic theory

Assumptions**:-**

- Plane section remains plane before and after deformation takes place
- Stress –strain relation under working load, is linear for both steel and concrete
- Tensile stress is taken care by reinforcement and none of them by concrete
- Modular Ratio between steel and concrete remains constant.

Modular ratio

$$m = \frac{Es}{Ec} = \frac{280}{\sigma_{cbc}}$$

Where σ_{cbc} = is permissible stress

Advantages:-

1. Method is simple
2. Method is reliable
3. Stress is very low under working condition , therefore serviceability is automatically satisfied

Limitations:-

1. Stress strain relation for concrete is not linear for concrete
2. It gives an idea that failure load = working load * factor of safety, but it is not true
3. This method gives uneconomical section

. Ultimate load method or Load factor method

- This method uses design load = ultimate load * load factor
- Load factor $= \frac{Collapse\ Load}{Working\ Load}$
- This method gives slender and thin section which results in excessive deflection and cracks
- This method does not take care of shrinkage of concrete
- This method does not take of serviceability

Limit State Method

Limit state is an acceptable limit for both safety and serviceability before which failure occurs

1. Limit state of collapse
2. Limit state of serviceability
3. Other limit state

Limit state of Collapse

The structure may get collapse because of

- Rupture at one or more cross-sections
- Buckling
- Overturning

While designing the structure following ultimate stresses should be considered

1. Flexure
2. Shear
3. Torsion
4. Tension
5. Compression

Limit state of Serviceability

a) Limit state of deflection

- Lack of safety
- Appearance
- Ponding of water
- Misalignment in machines
- Door, window frames, flooring materials undergoes crack

Methods for controlling deflecting

- Empherical formula – span/depth
- Theoretical - dimension

b) Limit state of cracking

- Appearance
- Lack of safety
- Leakage
- Creation of maintenance problem
- Reduction in stiffness with increase in deflection
- corrosion

Other Limit states

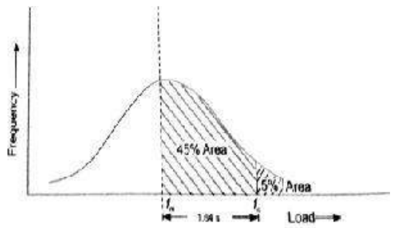
a) Vibration
b) Fire resistance
c) Chemical and environmental actions
d) Accidental loads

Types of Loads on RCC Structures

1. Dead Load IS 875 (Part 1)1987
2. Live Load IS 875 (Part 2)1987
3. Wind Load IS 875 (Part 3)1987
4. Snow Load IS 875 (Part 4)1987
5. Earthquake Load IS 1893 2002
   - Low intensity Zone (IV or less) – Zone II
   - Moderate intensity Zone (VII) – Zone III
   - Severe intensity Zone (VIII) – Zone IV
   - Very Severe intensity Zone (IX and above) – Zone V

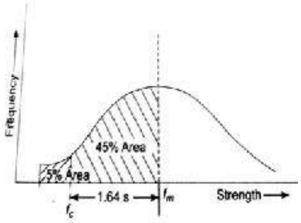
Characteristic load

Characteristic load = Mean Load+1.64S



Characteristic Strength

Characteristic Strength = Mean Strength -1.64S



Partial safety factor

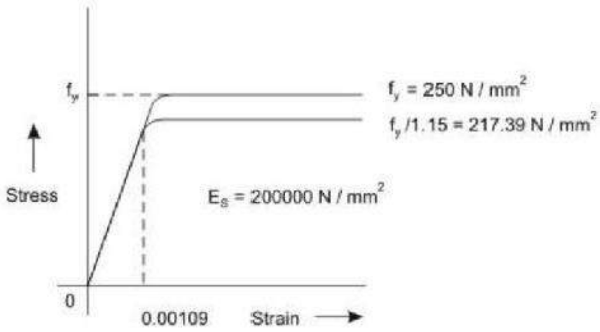
2. For material

$$f_d = \frac{f}{\gamma_m}$$

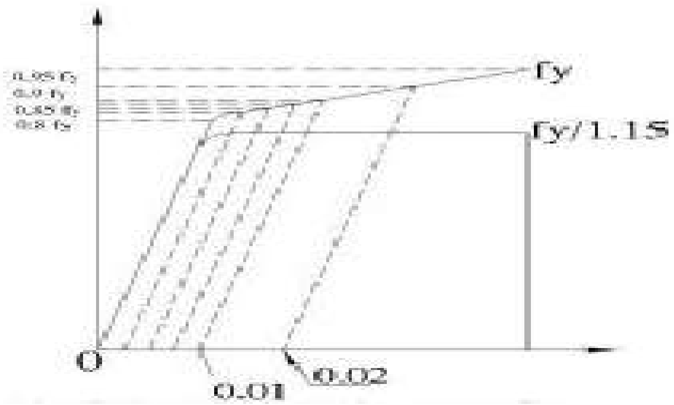
3. For load

$$F_d = F * \gamma_f$$

Stress-strain curves for reinforcement



Stress-strain curve for Mild steel (idealized) (Fe 250) with definite yield point



Stress-strain curve for cold worked deform bar

Figures show the representative stress-strain curves for steel having definite yield point and not having definite yield point, respectively. The characteristic yield strength *fy* of steel is assumed as the minimum yield stress or 0.2 per cent of proof stress for steel having no definite yield point. The modulus of elasticity of steel is taken to be 200000 N/mm^2

For mild steel, the stress is proportional to the strain up to the yield point. Thereafter, post yield strain increases faster while the stress is assumed to remain at constant value of *fy*.

Limit state of collapse in flexure

The behaviour of reinforced concrete beam sections at ultimate loads has been explained in detail in previous section. The basic assumptions involved in the analysis at the ultimate limit state of flexure (Cl. 38.1 of the Code) are listed here.

a) Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section.

b) The maximum compressive strain in concrete (at the outermost fibre) □ *cu* shall be taken as 0.0035 in bending.

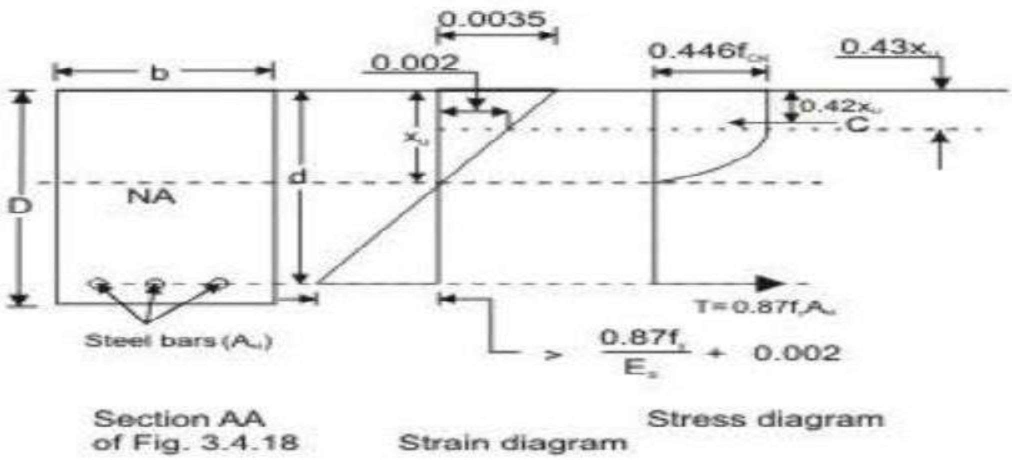
c) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress-strain curve is given below in figure 1.6. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor y, = 1.5 shall be applied in addition to this.

d) The tensile strength of the concrete is ignored.

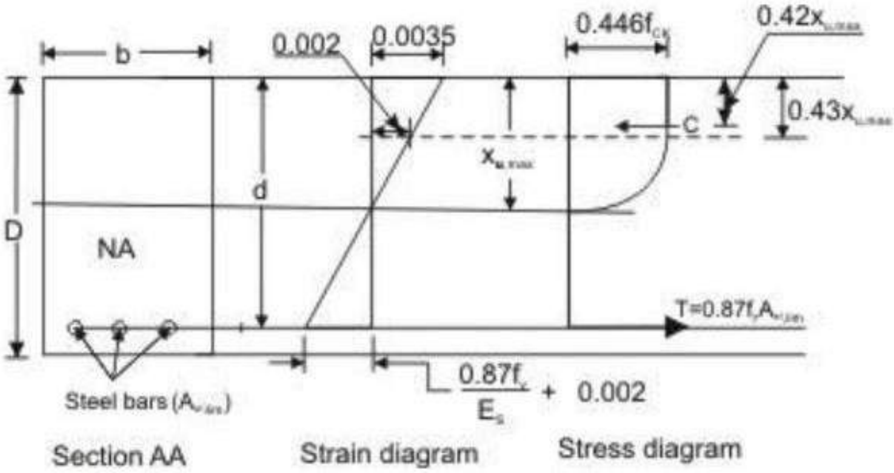
e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in figure 1.3. For design purposes the partial safety factor equal to 1.15 shall be applied.

f) The maximum strain in the tension reinforcement in the section at failure shall not be less than $\frac{f_y}{1.15E_s}+0.002$

Limiting Depth of Neutral Axis



Rectangular beam under flexure *xu* < *xu*max



Rectangular beam under flexure *xu* = *xu*,max

Based on the assumption given above, an expression for the depth of the neutral axis at the ultimate limit state, *xu* , can be easily obtained from the strain diagram in Fig

Considering similar triangles,

$$\frac{x_u}{d} = \frac{0.0035}{0.0035 + \frac{0.87 f_y}{E_s} + 0.002} \qquad (1)$$

According to IS 456:2000 cl no 38.1 (f) ,when the maximum strain in tension reinforcement is equal to $\frac{0.87 f_y}{E_s} + 0.002$, then the value of neutral axis will be $x_{u,\max}$.

Therefore, $$\frac{x_{u,\max}}{d} = \frac{0.0035}{0.0035 + \frac{0.87 f_y}{E_s} + 0.002} \qquad (2)$$

The values of *xu*,max for different grades of steel, obtained by applying Eq. (2), are listed in table.

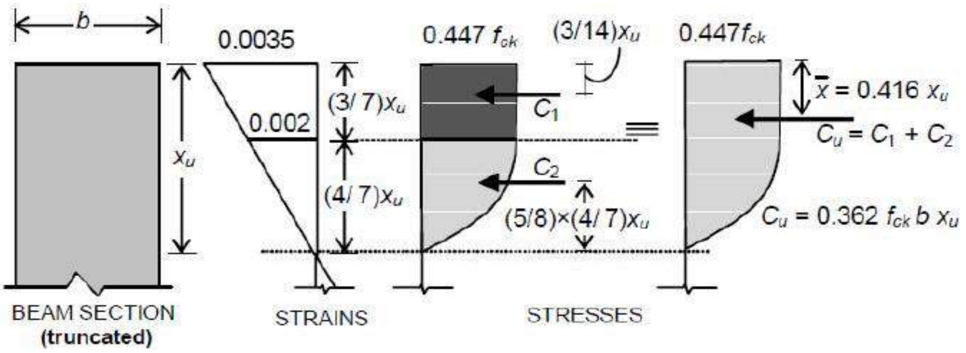
Limiting depth of neutral axis for different grades of steel

Steel Grade	Fe 250	Fe 415	Fe 500
$x_{u,\max}/d$	0.5313	0.4791	0.4791

The limiting depth of neutral axis *xu*,max corresponds to the so-called balanced section, i.e., a section that is expected to result in a balanced‘ failure at the ultimate limit state in flexure. If the neutral axis depth *xu* is less than *xu*,max , then the section is under-reinforced (resulting in a tension‘ failure); whereas if *xu* exceeds *xu*,max , it is over-reinforced (resulting in a compression‘ failure).

Analysis of Singly Reinforced Rectangular Sections

Analysis of a given reinforced concrete section at the ultimate limit state of flexure implies the determination of the *ultimate moment MR of resistance* of the section. This is easily obtained from the couple resulting from the flexural stresses



Concrete stress-block parameters in compression

$M_R = C * z = T * z$

where *C* and *T* are the resultant (ultimate) forces in compression and tension respectively and *z* is the lever arm.

$T = 0.87\ fy\ A_{st}$

Concrete Stress Block in Compression

In order to determine the magnitude of Cu and its line of action, it is necessary to analyze the concrete stress block in compression. As ultimate failure of a reinforced concrete beam in flexure occurs by the crushing of concrete, for both under- and over-reinforced beams, the shape of the compressive stress distribution (‗stress block‘) at failure will be, in both cases, as shown in Fig. The value of Cu can be computed knowing that the compressive stress in concrete is uniform at 0.447 fck for a depth of 3xu/ 7, and below this it varies parabolically over a depth of 4xu/ 7 to zero at the neutral axis.

For a rectangular section of width *b*,

$$C_u = 0.447 f_{ck}\, \mathrm{b}\left[\frac{3x_u}{7} + \left(\frac{2}{3} x \frac{4x_u}{7}\right)\right]$$

Therefore, Cu = 0.36 * fck * b * xu

Also, the line of action of *Cu* is determined by the centroid of the stress block, located at a distance *x* from the concrete fibres subjected to the maximum compressive strain.

Accordingly, considering moments of compressive forces $C\ u$, $C1$ and $C2$ about the maximum compressive strain location,

$$(0.362 f_{ck} b x_u) \text{x} \bar{x} - (0.447 f_{ck} b x_u)\left[\left(\frac{3}{7}\right)\left(\frac{1.5x_u}{7}\right)+\left(\frac{2}{3}\text{x}\frac{4}{7}\right)\left(x_u-\frac{5}{8}\text{x}\frac{4x_u}{7}\right)\right]$$

Solving $\bar{x} = 0.416x_u$

Depth of Neutral Axis

For any given section, the depth of the neutral axis should be such that $Cu = T$, satisfying equilibrium of forces.

Equating $C = T$,

$$x_u - \frac{0.87 f_y A_{st}}{0.361 f_{ck} b}$$, valid only if resulting $x_u \leq x_{u,\max}$

Ultimate Moment of Resistance

The ultimate moment of resistance MR of a given beam section is

Accordingly, in terms of the concrete compressive strength,

$M_{uR} = 0.361 f_{ck} b x_u (\text{d} - 0.416\text{x}_u)$ *for all* x_u

Alternatively, in terms of the steel tensile stress,

$M_{uR} = f_{st}\,\text{A}_{st}(\text{d} - 0.416\text{x}_u)$ *for all* x_u

With $f_{st}=0.87f_y$ for $x_u \leq x_{u,\max}$

Limiting Moment of Resistance

The *limiting moment of resistance* of a given (singly reinforced, rectangular) section, according to the Code (Cl. G–1.1), corresponds to the condition, defined by Eq. (2). From Eq. (9), it follows that:

$$M_{u,\lim} = 0.361 f_{ck} b x_{u,\max}(\text{d} - 0.416\text{x}_{u,\max}) \quad (11)$$

$$M_{u,\lim} = 0.361 f_{ck}\left(\frac{x_{u,\max}}{d}\right)\left(1-\frac{0.416\text{x}_{u,\max}}{d}\right)bd^2 \quad (11a)$$

Modes of failure: Types of section

A reinforced concrete member is considered to have failed when the strain of concrete in extreme compression fibre reaches its ultimate value of 0.0035. At this stage, the actual strain in steel can have the following values:

(a) Equal to failure strain of steel

(b) More than failure strain, corresponding to under reinforced section.

(c) Less than failure strain corresponding to over reinforced section.

Thus for a given section, the actual value of x_u / d can be determined from Eq. (7). Three cases arise.

Case-1: x_u /d equal to the limiting value $x_{u,max} /d$: Balanced section.

Case-2: x_u /d less than limiting value: under-reinforced section.

Case-3: x_u /d more than limiting value: over-reinforced section.

Balanced Section

In balanced section, the strain in steel and strain in concrete reach their maximum values simultaneously. The percentage of steel in this section is known as critical or limiting steel percentage. The depth of neutral axis (NA) is $x_u = x_{u,max}$.

Under-reinforced section

An under-reinforced section is the one in which steel percentage (pt) is less than critical or limiting percentage ($p_{t,lim}$). Due to this the actual NA is above the balanced NA and $x_u < x_{u,max}$.

Over-reinforced section

In the over reinforced section the steel percentage is more than limiting percentage due to which NA falls below the balanced NA and $x_u > x_{u,max}$. Because of higher percentage of steel, yield does not take place in steel and failure occurs when the strain in extreme fibres in concrete reaches its ultimate value.

General Aspects of Serviceability:

The members are designed to withstand safely all loads liable to act on it throughout its life using the limit state of collapse. These members designed should also satisfy the serviceability limit states. To satisfy the serviceability requirements the deflections and cracking in the member should not be excessive and shall be less than the permissible values. Apart from this the other limit states are that of the durability and vibrations. Excessive values beyond this limit state spoil the appearance of the structure and affect the partition walls, flooring etc. This will cause the user discomfort and the structure is said to be unfit for use.

The different load combinations and the corresponding partial safety factors to be used for the limit state of serviceability are given in Table 18 of IS 456:2000.

Load combination	Limit State of Collapse			Limit state of serviceability		
	DL	IL	WL	DL	IL	WL
DL + IL	1.5		1.0	1.0	1.0	-
DL + WL	1.5 or 0.9	-	1.5	1.0	-	1.0
DL + IL + WL	1.2			1.0	0.8	0.8

Limit state of serviceability for flexural members:

Deflection

The check for deflection is done through the following two methods specified by IS 456:2000 (Refer clause 42.1)

1 Empirical Method

In this method, the deflection criteria of the member is said to be satisfied when the actual value of span to depth ratio of the member is less than the permissible values. The IS code procedure for calculating the permissible values are as given below

a. Choosing the basic values of span to effective depth ratios (l/d) from the following, depending on the type of beam

1. Cantilever = 8
2. Simply supported = 20
3. Continuous = 26

b. Modify the value of basic span to depth ratio to get the allowable span to depth ratio.

Allowable l/d = Basic l/d x M_t x M_c x M_f

Where, M_t = Modification factor obtained from fig 4 IS 456:2000. It depends on the area of tension reinforcement provided and the type of steel.

M_c = Modification factor obtained from fig 5 IS 456:2000. This depends on the area of compression steel used.

M_f = Reduction factor got from fig 6 of IS 456:2000

Note: The basic values of l/d mentioned above is valid upto spans of 10m. The basic values are multiplied by 10 / span in meters except for cantilever. For cantilevers whose span exceeds 10 m the theoretical method shall be used.

2 Theoretical method of checking deflection

The actual deflections of the members are calculated as per procedure given in annexure 'C' of IS 456:2000. This deflection value shall be limited to the following

i. The final deflection due to all loads including the effects of temperature, creep and shrinkage shall not exceed span / 250.

ii. The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes shall not exceed span/350 or 20 mm whichever is less.

Cracking in structural members

Cracking of concrete occurs whenever the tensile stress developed is greater than the tensile strength of concrete. This happens due to large values of the following:

1. Flexural tensile stress because of excessive bending under the applied load
2. Diagonal tension due to shear and torsion
3. Direct tensile stress under applied loads (for example hoop tension in a circular tank)
4. Lateral tensile strains accompanying high axis compressive strains due to Poisson's effect (as in a compression test)
5. Settlement of supports

In addition to the above reasons, cracking also occurs because of

1. Restraint against volume changes due to shrinkage, temperature creep and chemical effects.
2. Bond and anchorage failures

Cracking spoils the aesthetics of the structure and also adversely affect the durability of the structure. Presence of wide cracks exposes the reinforcement to the atmosphere due to which the reinforcements get corroded causing the deterioration of concrete. In some cases, such as liquid retaining structures and pressure vessels cracks affects the basic functional requirement itself (such as water tightness in water tank).

Permissible crack width

The permissible crack width in structural concrete members depends on the type of structure and the exposure conditions. The permissible values are prescribed in clause 35.3.2 IS 456:2000 and are shown in table below

Table: Permissible values of crack width as per IS 456:2000

No.	Types of Exposure	Permissible widths of crack at surface (mm)
1	Protected and not exposed to aggressive environmental conditions	0.3
2	Moderate environmental conditions	0.2

Control of cracking

The check for cracking in beams are done through the following 2 methods specified in IS 456:2000 clause 43.1

1. By empirical method:

In this method, the cracking is said to be in control if proper detailing (i.e. spacing) of reinforcements as specified in clause 26.3.2 of IS 456:2000 is followed. These specifications regarding the spacing have been already discussed under heading general specifications. In addition, the following specifications shall also be considered

i. In the beams where the depth of the web exceeds 750 mm, side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall not be less than 0.1% of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less. (Refer clause 25.5.1.3 IS456:2000)

ii. The minimum tension reinforcement in beams to prevent failure in the tension zone by cracking of concrete is given by the following

As = 0.85 fy / 0.87 fy (Refer clause 26.5.1.1 IS 456:2000)

iii. Provide large number of smaller diameter bars rather than large diameter bars of the same area. This will make the bars well distributed in the tension zone and will reduce the width of the cracks.

2. By crack width computations In the case of special structures and in aggressive environmental conditions, it is preferred to compute the width of cracks and compare them with the permissible crack width to ensure the safety of the structure at the limit state of serviceability. The

IS 456-2000 has specified an analytical method for the estimation of surface crack width in Annexure-F which is based on the British Code (BS : 8110) specifications where the surface crack width is less than the permissible width, the crack control is said to be satisfied.

Problems:

1. Given the following data of a simply supported T beam, check the deflection criteria by empirical method

   Width of the beam (b) = 230 mm
   Effective depth (d) = 425 mm
   Effective span = 8.0 m
   Area of tension steel required = 977.5 mm^2
   Area of tension steel provided = 1256 mm^2
   Area of compression steel provided = 628 mm^2
   Type of steel = Fe 415
   Width of flange (b_f) = 0.9 m
   Width of web (b_w) = 0.3 m

   Solution:

   Basic $\frac{l}{d}$ = 20 for simply supported beam from clause 23.2.1

   Allowable $\frac{l}{d}$ = Basic $\frac{l}{d}$ x M_t x M_c x M_f

$$P_t = \frac{1265 \; x \; 100}{230 \; x \; 425} = 1.30 \;\%$$

$$f_s = \frac{0.58\ f_y\ \times\ Area\ of\ steel\ required}{Area\ of\ steel\ provided}$$

$$f_s = \frac{0.58\ \times\ 415\ \times\ 977.5}{1256} = 187.3$$

From fig 4, for $P_t = 1.3\%$, $f_s = 187.5\ N/mm^2$

M_t

$$P_c = \frac{628\ \times\ 100}{230\ \times\ 425} = 0.65\%$$

From fig 5, for $P_c = 0.65\%$, M_c

From fig 6, for $\frac{b_w}{b_f} = \frac{0.30}{0.90} = 0.33$, M_f

Substituting a, b and c in equation (1)

We get allowable $\frac{l}{d} = 20 \times 1.1 \times 1.15 \times 0.80 = 20.2$

Actual $\frac{l}{d} = \frac{8}{0.425} = 18.82 <$ allowable $\frac{l}{d}$

Hence OK

2. A rectangular beam continuous over several supports has a width of 300 mm and overall depth of 600 mm. The effective length of each of the spans of the beam is 12.0 m. The effective cover is 25 mm. Area of compression steel provided is 942 mm^2 and area of tension steel provided is 1560 mm^2. Adopting Fe 500 steel estimate the safety of the beam for deflection control using the empirical method

Solution:

Allowable $\frac{l}{d}$ = Basic $\frac{l}{d} \times M_t \times M_c \times M_f$

Basic $\frac{l}{d} = 26$ as the beam is continuous

$$f_s = \frac{0.58\ f_y\ \times\ Area\ of\ steel\ required}{Area\ of\ steel\ provided}$$

$$f_s = \frac{0.58\ \times\ 500\ \times\ 1560}{1560} = 290$$

From fig 4, for $f_s = 290$, $P_t = 0.90$, $M_t = 0.9$

From fig 5, for $P_c = 0.54\%$, M_c .(b)

From fig 6, for $\frac{b_w}{b_f} = 1.0$, M_f (c)

The equation (1) shall be multiplied by $\frac{10}{span}$ $i.e\ \frac{10}{12}$ as the span of the beam is greater than 10.0 m

Allowable $\frac{l}{d} = \frac{10}{12}$ x 26 x 0.9 x 1.15 x 1 = 22.4

Actual $\frac{l}{d} = \frac{12}{0.575} = 20.86 <$ allowable $\frac{l}{d}$

Hence deflection control is satisfied.

3. Find the effective depth based on the deflection criteria of a cantilever beam of 6m span. Take $f_y = 415$ N/mm^2, Pt = 1%, Pc = 1%.

Solution:

Allowable $\frac{l}{d}$ = Basic $\frac{l}{d}$ x M_t x M_c x M_f

Basic $\frac{l}{d} = 7$ for cantilever beam

Assume $\frac{A_{st}\ required}{A_{st}\ provided} = 1.0$

$f_s = 0.58 \times 415 \times 1 = 240.7$

From fig 4, for $f_s = 240$, $P_t = 1\%$, $M_t = 1.0$

From fig 5, for $P_c = 1\%$, $M_c = 1.25$

From fig 6, for $\frac{b_w}{b_f} = 1.0$, $M_f = 1$

Allowable $\frac{l}{d} = 7 \times 1.0 \times 1.25 \times 1.0 = 8.75$

$$d = \frac{l}{8.75} = \frac{6000}{8.75} = 685 \text{ mm}$$

4. A simply supported beam of rectangular cross section 250mm wide and 450mm overall depth is used over an effective span of 4.0m. The beam is reinforced with 3 bars of 20mm diameter Fe 415 HYSD bars at an effective depth of 400mm. Two anchor bars of 10mm diameter are provided. The self weight of the beam together with the dead load on the beam is 4 kN/m. Service load acting on the beam is 10 kN/m. Using M20 grade concrete, compute
   a. Short term deflection
   b. Long term deflection

Solution:

Data b = 250 mm, D = 450 mm, d = 400 mm, $f_y = 415$ N/mm^2

$A_{st} = 3 \times \frac{\pi}{4} \times 20^2 = 942$ mm^2, l = 4.0 m, D.L = 4 kN/m, Service load = 10 kN/m,

Total load = 14 kN/m, $f_{ck} = 20$, $A_{sc} = 2 \times \frac{\pi}{4} \times 10^2 = 158$ mm^2

$E_s = 2.1 \times 10^5$, Ec = $5000\sqrt{f_{ck}} = 22360$ N/mm^2

$$m = \frac{280}{3\,\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.3$$

$f_{cr} = 0.7\sqrt{f_{ck}} = 0.7\sqrt{20} = 3.13$ N/mm^2

a. Short term deflection

To determine the depth of N.A

Equating the moment of compression area to that of the tension area, we get

$$b * x * \frac{x}{2} = m * A_{st} * (d-x)$$

t the steel into equivalent concrete area

$$250 * \frac{x^2}{2} = 13 * 942 * (400-x)$$

Solving, x = 155 mm from the top

Cracked MOI $I_r = \frac{250 \times 155^3}{12} + (250 \times 155) \times (155/2)^2 + 13 \times 942\,(400 \quad 155)$

$= 10.45 \times 10^8$ mm^4

(2) I_{gr} = Gross MOI $= \frac{250 \times 450^3}{12} = 18.98 \times 10^8$ mm^4

(3) M = Maximum BM under service load

$$M = \frac{w\,l^2}{8} = \frac{14 \times 4^2}{8} = 28 \text{ kN} = 28 \times 10^6 \text{ N-mm}$$

(4) Cracked moment of inertia

$$M_r = \frac{f_{cr}\,I_{gr}}{y_t} = \frac{3.13 \times 18.98 \times 10^8}{0.5 \times 450} = 26 \times 10^6 \text{ N-mm}$$

Lever arm = z = $\left(d - \frac{x}{3}\right)$

$$= \left(400 - \frac{155}{3}\right) = 348.34 \text{ mm}$$

(5) $I_{eff} = \left[\frac{I_r}{1.2 - \left(\frac{m_r}{m}\right)\left(\frac{z}{d}\right)\left(1 - \frac{x}{d}\right)\left(\frac{b_w}{b}\right)}\right]$

$$\left[\frac{10.45 \times 10^8}{1.2 - \left(\frac{26 \times 10^6}{28 \times 10^6}\right)\left(\frac{348.34}{400}\right)\left(1 - \frac{155}{400}\right)(1)}\right]$$

$I_{eff} = 14.93 \times 10^8$ mm^4

Further $I_r < I_{eff} < I_{gr}$

(6) Maximum short term deflection

$$a_{i(perm)} = \frac{K_w\, w\, l^4}{E_c\, I_{eff}} = \frac{5}{384}\ \frac{14 \times (4000)^2}{22360 \times 14.93 \times 10^8} = 1.39 \text{ mm}$$

$K_w = \frac{5}{384}$ for SSB with UDL

b. Long term deflection

(1) Shrinkage deflection (a_{cs}):

$a_{cs} = K_3 \quad _{cs} L^2$

$K_3 = 0.125$ for simply supported beam from Annexure C-3.1

$_{cs}$ = Shrinkage curvature = $K_4\left(\frac{\epsilon_{cs}}{D}\right)$

ϵ_{cs} = Ultimate shrinkage strain of concrete (refer 6.2.4) = 0.0003

$$P_t = \frac{100 \times 942}{250 \times 400} = 0.942$$

$$P_c = \frac{100 \times 158}{250 \times 400} = 0.158$$

$P_t - P_c = (0.942 \quad 0.158) = 0.784$

$P_t - P_c = (0.942 \quad 0.158) = 0.784$

This is greater than 0.25 and less than 1.0 Hence ok.

Therefore $K_4 = 0.72 \times \frac{P_t - P_c}{\sqrt{P_t}} = 0.72 \times \frac{0.942 - 0.158}{\sqrt{0.942}}$

$K_4 = 0.58$

$_{cs} = \frac{0.58 \times 0.0003}{450} = 3.866 \times 10^{-7}$

$a_{cs} = K_3 \quad _{cs} L^2$

$= 0.125 \times 3.866 \times 10^{-7} \times (4000)^2$

$= 0.773$ mm

(2) Creep deflection [$a_{cc(perm)}$]

Creep deflection $a_{cc(perm)} = a_{icc(perm)} \quad a_{i(perm)}$

Where, $a_{cc(perm)}$ = creep deflection due to permanent loads

$a_{icc(perm)}$ = short term deflection + creep deflection

$a_{i(perm)}$ = short term deflection

$$a_{icc\ (perm)} = K_w \left(\frac{w\, l^3}{E_{ce}\, I_{eff}}\right)$$

$$E_{ce} = \frac{E_c}{(1+\theta)} = \frac{E_c}{(1+1.6)}$$

θ = Creep coefficient = 1.6 for 28 days loading

$a_{icc(perm)}$ = 2.6 x short term deflection

= 2.6 x $a_{i(perm)}$

= 2.6 x 1.39 = 3.614 mm

Creep deflection $a_{cc(perm)}$ = 3.614 1.39 = 2.224 mm

Total long term deflection = shrinkage deflection + Creep deflection

= 0.773 + 2.224 = 3.013 mm

Total deflection = Short term deflection + Long term deflection

= 1.39 + 3.013 = 4.402 mm

Outcome

1. Able to know various design philosophies.
2. Able to know the necessity of reinforcement in RC structure.
3. Able to know the stress block parameter of RC beam section.
4. Able to know the necessity of partial safety in design of RC member.

Assignment questions

1. What are the modes of failure of singly reinforced beam?
2. What are the methods of design philosophies?
3. What is moment of resistance?
4. What are the loads that are likely to act on the structure?
5. What is singly reinforced beam?

Future Study

https://nptel.ac.in/courses/105105105/

Module – 2

Analysis of Beam subjected to Flexure, shear and torsion

Introduction to failure modes of beams

Objectives

Shear Stress

Design shear strength of beam

Design of shear reinforcement

Bond strength

Development length

Outcome

Assignment questions

Future study

Introduction to Failure modes of beams

Failure Modes due to Shear

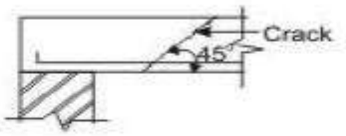


Figure 1.10 (a) Web shear progress along dotted lines

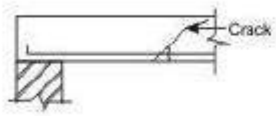


Figure 1.10 (b) Flexural tension

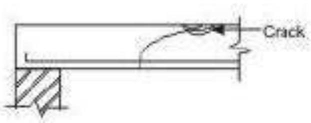


Figure 1.10 (b) Flexural compression

Bending in reinforced concrete beams is usually accompanied by shear, the exact analysis of which is very complex. However, experimental studies confirmed the following three different modes of failure due to possible combinations of shear force and bending moment at a given section:

(i) Web shear (Fig. 1.10a)

(ii) Flexural tension shear (Fig. 1.10b)

(iii) Flexural compression shear (Fig. 1.10c)

Web shear causes cracks which progress along the dotted line shown in Fig. 1.10a. Steel yields in flexural tension shear as shown in Fig. 1.10b, while concrete crushes in compression due to flexural compression shear as shown in Fig. 1.10c. An in-depth presentation of the three types of failure modes is beyond the scope here.

Objectives

1. To analyze the RCC beam as singly or doubly

Shear Stress

The distribution of shear stress in reinforced concrete rectangular, *T* and *L*-beams of uniform and varying depths depends on the distribution of the normal stress. However, for the sake of simplicity the nominal shear stress τ_v is considered which is calculated as follows (IS 456, cls. 40.1 and 40.1.1):

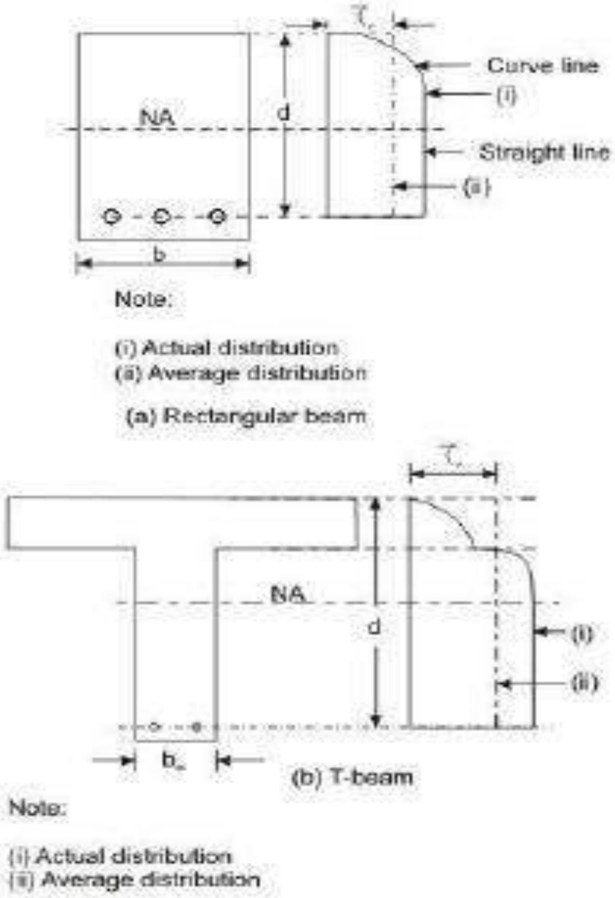


Figure 1.11: Distribution of shear stress and average shear stress

(i) In beams of uniform depth (Figs. 1.11a and b):

$$\tau_v = \frac{V_u}{bd}$$

where V_u = shear force due to design loads,

b = breadth of rectangular beams and breadth of the web b_w for flanged beams, and

Figure 1.11: Distribution of shear stress and average shear stress

(i) In beams of uniform depth (Figs. 1.11a and b):

$$\tau_v = \frac{V_u}{bd}$$

where V_u = shear force due to design loads,

b = breadth of rectangular beams and breadth of the web b_w for flanged beams, and

d = effective depth.

(ii) In beams of varying depth:

$$\tau_v = \frac{V_u \pm \frac{M_u}{d}\tan\beta}{bd}$$

where τ_v, Vu, b or b_w and d are the same as in (i),

M_u = bending moment at the section, and

β = angle between the top and the bottom edges.

The positive sign is applicable when the bending moment M_u decreases numerically in the same direction as the effective depth increases, and the negative sign is applicable when the bending moment M_u increases numerically in the same direction as the effective depth increases.

Design Shear Strength of Reinforced Concrete

Recent laboratory experiments confirmed that reinforced concrete in beams has shear strength even without any shear reinforcement. This shear strength (τ_c) depends on the grade of concrete and the percentage of tension steel in beams. On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some maximum value τ_{cmax} depending on the grade of concrete. These minimum and maximum shear strengths of reinforced concrete (IS 456, cls. 40.2.1 and 40.2.3, respectively) are given below:

Design shear strength without shear reinforcement (IS 456, cl. 40.2.1)

Table 19 of IS 456 stipulates the design shear strength of concrete τ_c for different grades of concrete with a wide range of percentages of positive tensile steel reinforcement. It is worth mentioning that the reinforced concrete beams must be provided with the minimum shear reinforcement as per cl. 40.3 even when τ_v is less than τ_c

Minimum Shear Reinforcement (cls. 40.3, 26.5.1.5 and 26.5.1.6 of IS 456)

Minimum shear reinforcement has to be provided even when τ_v is less than τ_c given in Table 3 as recommended in cl. 40.3 of IS 456. The amount of minimum shear reinforcement, as given in cl. 26.5.1.6, is given below.

The minimum shear reinforcement in the form of stirrups shall be provided such that:

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y} \qquad (15)$$

where A_{sv} = total cross-sectional area of stirrup legs effective in shear,

s_v = stirrup spacing along the length of the member,

b = breadth of the beam or breadth of the web of the web of flanged beam b_w, and

f_y = characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be taken greater than 415 N/mm^2.

The above provision is not applicable for members of minor structural importance such as lintels where the maximum shear stress calculated is less than half the permissible value.

The minimum shear reinforcement is provided for the following:

(i) Any sudden failure of beams is prevented if concrete cover bursts and the bond to the tension steel is lost.

(ii) Brittle shear failure is arrested which would have occurred without shear reinforcement.

(iii) Tension failure is prevented which would have occurred due to shrinkage, thermal stresses and internal cracking in beams.

(iv) To hold the reinforcement in place when concrete is poured.

(v) Section becomes effective with the tie effect of the compression steel.

Further, cl. 26.5.1.5 of IS 456 stipulates that the maximum spacing of shear reinforcement measured along the axis of the member shall not be more than 0.75 d for vertical stirrups and d for inclined stirrups at 45^o, where d is the effective depth of the section. However, the spacing shall not exceed 300 mm in any case.

Design of Shear Reinforcement (cl. 40.4 of IS 456)

When τ_v is more than τ_c given in Table 6.1, shear reinforcement shall be provided in any of the three following forms:

(a) Vertical stirrups,

(b) Bent-up bars along with stirrups, and

(c) Inclined stirrups.

In the case of bent-up bars, it is to be seen that the contribution towards shear resistance of bent-up bars should not be more than fifty per cent of that of the total shear reinforcement. The amount of shear reinforcement to be provided is determined to carry a shear force V_{us} equal to

$$V_{us} = V_u - \tau_c bd \qquad (16)$$

where b is the breadth of rectangular beams or b_w in the case of flanged beams.

The strengths of shear reinforcement V_{us} for the three types of shear reinforcement are as follows:

(a) Vertical stirrups:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v} \qquad (17)$$

(b) For inclined stirrups or a series of bars bent-up at different cross-sections:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}(\sin\alpha + \cos\alpha) \qquad (18)$$

(c) For single bar or single group of parallel bars, all bent-up at the same cross-section:

$$V_{us} = 0.87 f_y A_{sv} d \sin\alpha \qquad (19)$$

where A_{sv} = total cross-sectional area of stirrup legs or bent-up bars within a distance s_v,

s_v = spacing of stirrups or bent-up bars along the length of the member,

τ_v = nominal shear stress,

τ_c = design shear strength of concrete,

b = breadth of the member which for the flanged beams shall be taken as the breadth of the web b_w,

f_y = characteristic strength of the stirrup or bent-up reinforcement which shall not be taken greater than 415 N/mm^2,

α = angle between the inclined stirrup or bent-up bar and the axis of the member, not less than 45^o, and

d = effective depth.

The following two points are to be noted:

(i) The total shear resistance shall be computed as the sum of the resistance for the various types separately where more than one type of shear reinforcement is used.

(ii) The area of stirrups shall not be less than the minimum specified in cl. 26.5.1.6.

Numerical Problem

Find the moment of resistance of a singly reinforced concrete beam of 200 mm width 400mm effective depth, reinforced with 3-16 mm diameter bars of Fe 415 steel. Take M20 grade of concrete.

Solution

$$A_{st} = 3x\frac{\pi}{4}(16)^2 = 603.19\,mm^2$$

$$\%\,p_t = 100x\frac{603.19}{200x400} = 0.754\%$$

$$\frac{x_u}{d} = 2.417p_t\frac{f_y}{f_{ck}} = 2.417x\frac{0.754}{100}x\frac{415}{20} = 0.378$$

Now for Fe 415 grade of steel, $\frac{x_{u,\max}}{d} = 0.479$

Hence the beam is under-reinforced.

The moment of resistance is given by

$$M_u = 0.87f_yA_{st}d\left(1-\frac{f_yA_{st}}{f_{ck}bd}\right)$$

$$= 0.87x415x603.19x400\left(1-\frac{415x603.19}{20x200x400}\right)$$

=73.48 KN-m.

Bond

The bond between steel and concrete is very important and essential so that they can act together without any slip in a loaded structure. With the perfect bond between them, the plane section of a beam remains plane even after bending. The length of a member required to develop the full bond is called the anchorage length. The bond is measured by bond stress.

The local bond stress varies along a member with the variation of bending moment.

Thus, a tensile member has to be anchored properly by providing additional length on either side of the point of maximum tension, which is known as Development length in tension‘.

Similarly, for compression members also, we have Development length *Ld* in compression‘.

Accordingly, IS 456, cl. 26.2 stipulates the requirements of proper anchorage of reinforcement in terms of development length *Ld* only employing design bond stress η*bd*

.

Design bond stress – values

The average bond stress is still used in the working stress method and IS 456 has mentioned about it in cl. B-2.1.2. However, in the limit state method of design, the average bond stress has been designated as design bond stress η*bd* and the values are given in cl. 26.2.1.1

Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
Design Bond Stress τ_{bd} in N/mm2	1.2	1.4	1.5	1.7	1.9

For deformed bars conforming to IS 1786, these values shall be increased by 60 per cent. For bars in compression, the values of bond stress in tension shall be increased by 25 per cent.

Development Length

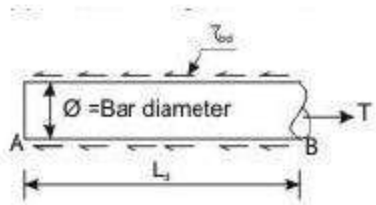


Figure 1.13 Development length of bar

Figure 1.13 shows the free body diagram of the segment AB of the bar. At B, the tensile force *T* trying to pull out the bar is of the value *T* = (π θ2 ζs /4), where θ is the nominal diameter of the bar and ζs is the tensile stress in bar at the section considered at design loads. It is necessary to have the resistance force to be developed by η*bd* for the length *Ld* to overcome the tensile force. The resistance force = π θ (*Ld*) (η*bd*). Equating the two, we get π θ (*Ld*) (η*bd*) = (π θ2 ζs/4) (19)

Equation (19), thus gives

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} \tag{20}$$

The above equation is given in cl. 26.2.1 of IS 456 to determine the development length of bars.

The example taken above considers round bar in tension. Similarly, other sections of the bar should have the required L_d as determined for such sections. For bars in compression, the development length is reduced by 25 per cent as the design bond stress in compression τ_{bd} is 25 per cent more than that in tension (see the last lines below Table 6.4). Following the same logic, the development length of deformed bars is reduced by 60 per cent of that needed for the plain round bars. Tables 64 to 66 of SP-16 present the development lengths of fully stressed plain and deformed bars (when $\sigma_s = 0.87 f_y$) both under tension and compression. It is to be noted that the consequence of stress concentration at the lugs of deformed bars has not been taken into consideration.

Outcome

1. Able to analyze singly and doubly reinforced beam
2. Able to know failure modes of beams
3. Able to know the shear behavior of beams

Assignment questions

1. What is the difference is between singly reinforced and doubly reinforced beam?
2. Explain different types of stirrups with a neat sketch.
3. Describe the failure modes of beams with a neat sketch.
4. What is development length?

Future Study

https://nptel.ac.in/courses/105105104/pdf/m5l11.pdf

Module – 3

Design of Beams

Introduction to beams

Objective

Types of beams

Design procedure

Problems

Outcome

Assignment questions

Future study

Introduction

Beam is a structural member which is normally placed horizontally. It provides resistance to bending when loads are applied on it. Most commonly used material for beam is RCC (Reinforced Cement Concrete). RCC beam can be various types depending on different criteria.

RCC beam can be various types depending on different criteria. Such as depending on shape, beam can be rectangular, T-beam etc. Depending on reinforcement placement, beam can be double reinforced beam, single reinforced beam, etc.

Objective

1. To design singly and doubly reinforced beam

Types of RCC Beams

RCC beams are 4 types depending on their supporting systems.

1. Simply supported beam
2. Semi-continuous beam
3. Continuous beam, and
4. Cantilever beam.

Simply Supported Beam

This type of beam has a single span. It is supported by two supports at both ends. This beam is also called simple beam.

Semi-Continuous Beam

This beam doesn’t have more than two spans. And supports are not more than three. Technically this beam is a continuous beam.

Continuous Beam

This type of beam has more than two spans and has more than three supports along its length. The supports are in one straight line thus the spans are also in a straight line.

Cantilever Beam

It has only one support in one end, another end is open.

There is another type of beam we can see in the civil engineering world which is called over-hanging beam. This beam extends beyond its supports. Actually this beam is a combination of simply supported and cantilever beam.

In this chapter, it is intended to learn the method of designing the beams using the principles developed in previous chapters. Design consists of selecting proper materials, shape and size of the structural member keeping in view the economy, stability and aesthetics. The design of beams are done for the limit state of collapse and checked for the other limit states. Normally the beam is designed for flexure and checked for shear, deflection, cracking and bond.

Design procedure

The procedure for the design of beam may be summarized as follows:

1. Estimation of loads
2. Analysis
3. Design

1. Estimation of loads

The loads that get realized on the beams consist of the following:

a. Self weight of the beam.

b. Weight of the wall constructed on the beam

c. The portion of the slab loads which gets transferred to the beams. These slab loads are due to live loads that are acting on the slab dead loads such as self weight of the slab, floor finishes, partitions, false ceiling and some special fixed loads. The economy and safety of the beams achieved depends on the accuracy with which the loads are estimated.

The dead loads are calculated based on the density whereas the live loads are taken from IS: 875 depending on the functional use of the building.

2. Analysis

For the loads that are acting on the beams, the analysis is done by any standard method to obtain the shear forces and bending moments.

3. Design

a. Selection of width and depth of the beam.

The width of the beam selected shall satisfy the slenderness limits specified in IS 456 : 2000 clause 23.3 to ensure the lateral stability.

b. Calculation of effective span (le) (Refer clause 22.2, IS 456:2000)

c. Calculation of loads (w)

d. Calculation of critical moments and shears.

The moment and shear that exists at the critical sections are considered for the design. Critical sections are the sections where the values are maximum. Critical section for the moment in a simply supported beam is at the point where the shear force is zero. For continuous beams the critical section for the +ve bending moment is in the span and –ve bending moment is at the support. The critical section for the shear is at the support.

e. Find the factored shear (Vu) and factored moment (Mu)

f. Check for the depth based on maximum bending moment.

Considering the section to be nearly balanced section and using the equation

Annexure G, IS 456-2000 obtain the value of the required depth drequired. If the assumed depth “d” is greater than the “drequired”, it satisfies the depth criteria based on flexure. If the assumed section is less than the” drequired”, revise the section.

g. Calculation of steel.

As the section is under reinforced, use the equation G.1.1.(b) to obtain the steel.

h. Check for shear.

i. Check for developmental length.

j. Check for deflection.

k. Check for Ast min, Ast max and distance between the two bars.

Anchorage of bars or check for development length

In accordance with clause 26.2 IS 456: 2000, the bars shall be extended (or anchored) for a certain distance on either side of the point of maximum bending moment where there is maximum stress (Tension or Compression). This distance is known as the development length and is required in order to prevent the bar from pulling out under tension or pushing in under compression. The development length (Ld) is given by

$$L_d = \frac{\emptyset\, \sigma_s}{4\, Z_{bd}}$$

where, Ø = Nominal diameter of the bar

σ_s – Stress in bar at the section considered at design load

Z_{bd} – Design bond stress given in table 26.2.1.1 (IS 456 : 2000)

Table 26.2.1.1: Design bond stress in limit state method for plain bars in tension shall be as below:

Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
Design bond stress τ_{bd} N/mm^2	1.2	1.4	1.5	1.7	1.9

Note: Due to the above requirement it can be concluded that no bar can be bent up or curtailed upto a distance of development length from the point of maximum moment.

Due to practical difficulties if it is not possible to provide the required embedment or development length, bends hooks and mechanical anchorages are used.

Flexural reinforcement shall not be terminated in a tension zone unless any one of the following condition is satisfied:

a. The shear at the cut-off points does not exceed two-thirds that permitted, including the shear strength of web reinforcement provided.

b. Stirrup area in excess of that required for shear and torsion is provided along each terminated bar over a distance from the cut-off point equal to three-fourths the effective depth of the member. The excess stirrup area shall be not less than $0.4bs/f_y$, where b is the breadth of the beam, s is the spacing and f_y is the characteristic strength of reinforcement in N/mm^2. The resulting spacing shall not exceed d/8 where is the ratio of the area of bars cut-off to the total area of bars at the section, and d is the effective depth.

c. For 36 mm and smaller bars, the continuing bars provide double the area required for flexure at the cut-off point and the shear does not exceed three-fourths that permitted.

Positive moment reinforcement:

a. At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of the member into the support, to a length equal to $L_d/3$.

b. When a flexural member is part of the primary lateral load resisting system, the positive reinforcement required to be extended into the support as described in (a) shall be anchored to develop its design stress in tension at the face of the support.

c. At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that L_d computed for f_d by 26.2.1

IS 456:2000 does not exceed.

$$\frac{M_1}{V} + L_0$$

Where, M_1 = moment of resistance of the section assuming all reinforcement at the section to be stressed to f_d;

$f_d = 0.87f_y$ in the case of limit state design and the permissible stress in the case of working stress design;

V = shear force at the section due to the design loads;

L_0 = sum of the anchorage beyond the centre of the support and the equivalent anchorage value of any hook or mechanical anchorage at simple support; and at a point of inflection, L_0 is limited to the effective depth of the members or 12 , whichever is greater; and

ϕ= diameter of bar.

The value of M_1/V in the above expression may be increased by 30 percent when the ends of the reinforcement are confined by a compressive reaction.

Negative moment reinforcement:

At least one third of the total reinforcement provided for negative moment at the support shall extend beyond the point of inflection for a distance not less than the effective depth of the member of 12 or one-sixteenth of the clear span whichever is greater.

Anchorage of bars

Anchoring of bars is done to provide the development length and maintain the integrity of the structure.

Anchoring bars in tension:

a. Deformed bars may be used without end anchorages provided development length requirement is satisfied. Hooks should normally be provided for plain bars in tension.

b. Bends and hooks – shall conform to IS 2502

1. Bends – The anchorage value of bend shall be taken as 4 times the diameter of the bar for each 450 bend subject to a maximum of 16 times the diameter of the bar.

2. Hooks – The anchorage value of a standard U-type hook shall be equal to 16 times the diameter of the bar.

Anchoring bars in compression:

The anchorage length of straight bar in compression shall be equal to the development length of bars in compression as specified in clause 26.2.1 of IS 456:2000. The projected length of hooks, bends and straight lengths beyond bends if provided for a bar in compression, shall only be considered for development length.

Mechanical devices for anchorage:

Any mechanical or other device capable of developing the strength of the bar without damage to concrete may be used as anchorage with the approval of the engineer-in-charge.

Anchoring shear reinforcement:

a. Inclined bars – The development length shall be as for bars in tension; this length shall be measured as under:

1. In tension zone, from the end of the sloping or inclined portion of the bar, and

2. In the compression zone, from the mid depth of the beam.

b. Stirrups – Not withstanding any of the provisions of this standard, in case of secondary reinforcement, such as stirrups and transverse ties, complete development lengths and anchorages shall be deemed to have been provided when the bar is bent through an angle of at least 900 round a bar of at least its own diameter and is continued beyond the end of the curve for a length of at least eight diameters, or when the bar is bent through an angle of 1350 and is continued beyond the end of the curve for a length of at least six bar diameters or when the bar is bent through an angle of 1800 and is continued beyond the end of the curve for a length of at least four bar diameters.

Reinforcement requirements

1. Minimum reinforcement:

The minimum area of tension reinforcement shall be not less than that given by the following:

Where,

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

Where, A_s = minimum area of tension reinforcement.

b = breadth of beam or the breadth of the web of T-beam,

d = effective depth, and

f_y = characteristic strength of reinforcement in N/mm2

2. Maximum reinforcement – The maximum area of tension reinforcement shall not exceed 0.04bD

Compression reinforcement:

The maximum area of compression reinforcement shall not exceed 0.04bD.

Compression reinforcement in beams shall be enclosed by stirrups for effective lateral restraint.

Pitch and diameter of lateral ties:

The pitch of shear reinforcement shall be not more than the least of the following distances:

1. The least lateral dimension of the compression members;
2. Sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and
3. 300 mm.

The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 16 mm.

Slenderness limits of beams to ensure lateral stability

A beam is usually a vertical load carrying member. However, if the length of the beam is very large it may bend laterally. To ensure lateral stability of a beam the following specifications have been given in the code.

A simply supported or continuous beam shall be so proportioned that the clear distance between the lateral restraints does not exceed 60b or whichever is less, where d is the effective depth of the beam and b the breadth of the compression face midway between the lateral restraints.

For a cantilever, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed 25b or whichever is less.

Problems:

1. Design a singly reinforced SSB of clear span 5m to support a working live load of 25 kN/m run. Use Fe 415 steel and M 20 grad concrete. Assume the support thickness as 230 mm.

Step 1 (a): Fixing up the depth of the section.

Taking $\frac{L}{d} = 20$, for SSB [Refer 23.2.1, pg 37]

$$d = \frac{L}{20} = \frac{5}{20} = 0.25 \text{ m} = 250 \text{ mm}$$

Providing a cover of 25 mm, overall depth D = 250 + 25 = 275 mm

Dimensions of the section.

Width b = 230 mm

depth d = 250 mm

Step 1 (b): Check for lateral stability/lateral buckling

Refer page 39, clause 23.3

Allowable l = 60b or $\frac{250\,b^2}{d}$

Allowable l = 60b = 13800 mm = 13.8 m

Or $\frac{250\,b^2}{d}$ = 52900 mm = 52.9 m

Allowable l = Lesser of the two values

= 13.8 m

Actual l of the beam (5m) < Allowable value of l. Hence ok

Or $\frac{100\,b^2}{d}$ = 11750 mm = 11.75 m

Allowable l = Lesser of the two values

= 5.75 m

Actual l of the beam (5m) < Allowable value of l.

Hence ok

Step 2: Effective span

Referring class 22.2 page 34,

Effective span l_e = clear span + d

Or l_e = clear span + $\frac{1}{2}$ support thickness

= clear span + $\frac{l_s}{2}$

Whichever is lesser.

L_e = 2 m + 450 mm = 2450 mm

Or L_e = 2 m + $\frac{230}{2}$ = 2115 mm

Therefore l_e = 2115 mm

Step 3: Calculation of loads

Consider 1m length of the beam

a. Dead load = $(0.23 \times 0.475 \times 1m \times 25\ kN/m^3) \times 1.5 = 4.096 \approx 4.1$ kN/m
b. Factored live load = 30 kN/m

Total Factored load W_u = 34.1 kN/m ≈ 35 kN/m

Factored moment $M_u = \frac{W_u \times l_e^2}{2} = \frac{35 \times 2.115^2}{2} = 78.28$ kN-m

Factored shear = 35 x 2.115 = 74.025 kN

Step 4: Check for depth based on flexure or bending moment consideration

Assuming the section to be nearly balanced, and equating M_u to M_{ulim},

$M_u = M_{ulim}$ = 78.28 kN-m

Using the equation G 1.1 (c)

$$M_{ulim} = 0.36 \frac{x_{umax}}{d} \left(1 - 0.42 \frac{x_{umax}}{d}\right) b d^2 f_{ck}$$

$$78.28 \times 10^6 = 0.36 \times 0.48\,(1 - 0.42 \times 0.48) 230 d^2 \times 20$$

d = 222 mm

$d_{assumed} > d_{required}$

Hence ok.

Step 5: Calculation of steel

Since the section is under reinforced we have,

Using equation G 1.1 (b)

$$M_u = 0.87 f_y A_{st}\, d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$78.28 \times 10^6 = 0.87 \times 415 \times A_{st} \times 450 \left(1 - \frac{A_{st} \times 415}{230 \times 450 \times 20}\right)$$

Solving the quadratic equation, $A_{st} = 540.33\ mm^2 \approx 540\ mm^2$

Choosing 16 mm diameter bars,

$$\text{Area of 1 bar} = \frac{\pi}{4} \times 16^2 = 201.06 \text{ mm}^2$$

Therefore number of bars of 8mm required = 2.69= 3 bars

Distance between any two bars

Minimum distance between two bars is greater of the following:

a. Size of the aggregate + 5 mm
   20 mm + 5 mm
b. Size of the bar (whichever is greater)=16mm

Therefore minimum distance = 25 mm

$$\text{Distance between the bars} = \frac{230-2\times25-2\times16-2\times8}{2} = 58\text{mm}$$

Distance provided = 58mm > Minimum distance 25mm

Hence ok.

Check for $A_{st\,min}$

$$A_{st\,min} = \frac{0.85bd}{0.87f_y}$$

$$A_{st\,min} = \frac{0.85 \times 230 \times 450}{0.87 \times 415} = 243.66 \text{ mm}^2$$

A_{st} provided = 3 x $\frac{\pi}{4}$ x 16^2 = 603.18 mm^2 > $A_{st\,min}$

Hence ok.

Check for $A_{st\,max}$

$A_{st\,max}$ = 0.04 x b x D = 4370 mm^2

A_{st} provided = 603.18 mm^2

$$A_{st\,min} < A_{st} < A_{st\,max}$$

Hence ok.

Check for shear

V_u = 74.025 kN

$\tau_v = \frac{V_u}{bd}$ = 0.715 N/mm^2

$P_t = \frac{100A_{st}}{bd} = \frac{100\times603.18}{230\times450} = 0.58$

From table 19,

τ_c = 0.51 N/mm^2

From table 20,

$\tau_{c\,max}$ − 2.8 N/mm^2

$\tau_c < \tau_v < \tau_{c\,max}$

Hence design of shear reinforcement is required

Selecting 2 leg vertical stirrups of 8 mm diameter, Fe 415 steel,

$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100$ mm^2

V_c = Shear force taken up by the concrete

$= \frac{\tau_c bd}{1000} = \frac{0.51 \times 230 \times 450}{1000} = 52.78$ kN

$V_u = 74.025$ kN

$V_{us} = V_u - V_c$

$= 74.025 - 52.785 = 21.24$ kN

$V_{us} = \frac{0.87 \times f_y \times A_{sv} \times d}{S_v}$

$21.24 \times 10^3 = \frac{0.87 \times 415 \times 100 \times 450}{S_v}$

$S_v = 764$ mm

Check for maximum spacing

Maximum spacing = 0.75d or 300mm whichever is lesser

Maximum spacing = 337.5 or 300mm

Therefore maximum spacing allowed = 300mm

Let us provide 8 mm dia 2-leg vertical stirrups at a spacing of 300 mm.

Check for $A_{sv\ min}$:

A_{sv} provided = 100 mm^2

$A_{sv\ min} = \frac{0.4 b S_v}{0.87 f_y} = 76.44$ mm^2

A_{sv} provided > $A_{sv\ min}$

Hence ok.

Check for deflection:

Allowable $\frac{l}{d}$ = Basic $\frac{l}{d}$ x M_t x M_c x M_f

Basic $\frac{l}{d} = 7$ as the beam is cantilever

From fig 4, $M_t = 1.2$

From fig 5, $M_c = 1$

From fig 6, $\frac{b_w}{b_f} = 1$ [Since it is rectangular section $b_w = b_f$]

Therefore allowable l/d = 7 x 1.2 x 1 x 1 = 8.4

Actual l/d = $\frac{2115}{450} = 4.7$ < Allowable l/d. Hence ok.

3. Design a reinforced concrete beam of rectangular section using the following data:

Effective span	= 5 m
Width of beam	= 250 mm
Overall depth	= 500 mm
Service load (DL+LL)	= 40 kN/m
Effective cover	= 50 mm
Materials	: M20 grade concrete and Fe 415 steel

a. Data

b = 250 mm	f_{ck} = 20 N/mm^2
D = 500 mm	f_y = 415 N/mm^2
d = 450 mm	E_s = 2 x 10^5 N/mm^2
d' = 50 mm	
l_e = 5 m	

w = 40 kN/m and W_u = 40 x 1.5 =60 kN/m

b. Ultimate moments and shear forces

$M_u = \frac{W_u \times l_e^2}{8} = \frac{60 \times 5^2}{8} = 187.5$ kN-m

V_u = Factored shear = $\frac{W_u \times l_e}{2} = 150$ kN

c. Determination of M_{ulim} and f_{sc}

$$M_{ulim} = 0.36\frac{x_{umax}}{d}\left(1 - 0.42\frac{x_{umax}}{d}\right)bd^2f_{ck}$$

$$M_{ulim} = 0.36 \times 0.48\,(1 - 0.42 \times 0.48)250 \times 450^2 \times 20$$

= 140 kN.m

Since $M_u > M_{u\,lim}$, design a doubly reinforced section

$(M_u - M_{u\,lim})$ = 187.5-140 = 47.5 kN.m

$$f_{sc} = \epsilon_{sc} \times E_s$$

Where, $\epsilon_{sc} = \left\{\frac{0.0035(x_{u\,max} - d')}{x_{u\,max}}\right\}$

$f_{sc} = \left\{\frac{0.0035(x_{u\,max} - d')}{x_{u\,max}}\right\} E_s$

$= \left\{\frac{0.0035[(0.48 \times 450) - 50]}{0.48 \times 450}\right\} 2 \times 10^5$

= 538 N/mm^2

But $f_{sc} \not> 0.87f_y - (0.87 \times 415) - 361$ N/mm^2

Therefore f_{sc} = 361 N/mm^2

steel $A_{sc} = \left[\frac{(M_u - M_{u\,lim})}{f_{sc}(d - d')}\right]$

$= \left[\frac{(47.5 \times 10^6)}{361 \times 400}\right] = 329$ mm^2

Provide 2 bars of 16mm diameter (A_{sc} = 402 mm^2)

$$A_{st2} - \left(\frac{A_{sc}f_{sc}}{0.87f_y}\right) - \left(\frac{329 \times 361}{0.87 \times 415}\right) = 329 \text{ mm}^2$$

$$A_{st1} = \left[\frac{0.36f_{ck}bx_{u\,lim}}{0.87f_y}\right]$$

$$= \left[\frac{0.36 \times 20 \times 250 \times 0.48 \times 450}{0.87 \times 415}\right] = 1077 \text{ mm}^2$$

Total tension reinforcement = $A_{st} = (A_{st1} + A_{st2})$

$= (1077 + 329)$

$= 1406\ mm^2$

Provide 3 bars of 25mm diameter ($A_{st} = 1473\ mm^2$)

d. Shear reinforcements

$\tau_v = (V_u/bd) = (150 \times 10^3) / (250 \times 450) = 1.33\ N/mm^2$

$$P_t = \frac{(100A_s)}{bd} = \frac{100 \times 1473}{250 \times 450} = 1.3$$

Referring table 19 of IS : 456 – 2000 ,

$\tau_c = 0.68\ N/mm^2$

$\tau_{cmax} = 2.8\ N/mm^2$ for M20 concrete from table 20 of IS 456-2000

Since $\tau_c < \tau_v < \tau_{cmax}$, shear reinforcements are required.

$V_{us} = [V_u - (\tau_c bd)]$

$= [150-(0.68 \times 250 \times 450)10^{-3}] = 73.5\ kN$

Using 8 mm diameter 2 legged stirrups,

$$S_v = \frac{0.87 \times f_y \times A_{sv} \times d}{V_{us}} = \frac{0.87 \times 415 \times 2 \times 50 \times 450}{73.5 \times 10^3} = 221\text{mm}$$

Maximum spacing is 0.75d or 300 mm whichever is less

$S_v > 0.75d = (0.75 \times 450) = 337.5$ mm

Adopt a spacing of 200 mm near supports gradually increasing to 300 mm towards the centre of the span.

e. Check for deflection control

$(l/d)_{actual} = (5000/450) = 11.1$

$(l/d)_{allowable} = [(l/d)_{basic} \times M_t \times M_c \times M_f]$

$P_t = 1.3$ and $P_c = [(100 \times 402) / (250 \times 450)] = 0.35$

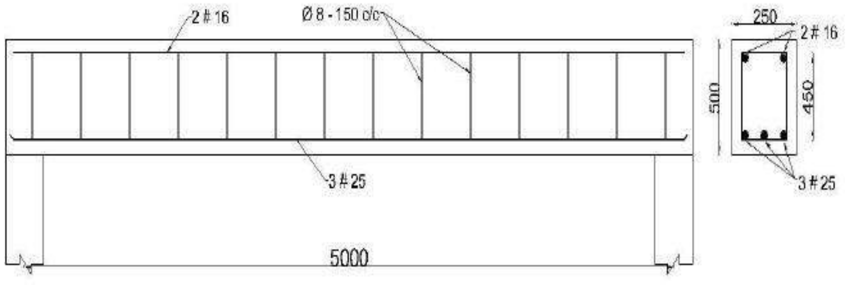
Refer Fig 4, $M_t = 0.93$
Fig 5, $M_c = 1.10$
Fig 6, $M_f = 1.0$

$(l/d)_{allowable} = [(20 \times 0.93 \times 1.10 \times 1] = 20.46$

$(l/d)_{actual} < (l/d)_{allowable}$

Hence deflection control is satisfied.

f. Reinforcement details



Outcome

1. Able to design singly and doubly reinforced beam

Assignment questions

A tee beam slab floor of an office comprises of a slab 150 mm thick spanning between ribs spaced at 3m centres. The effective span of the beam is 8 m. Live load on floor is 4 kN/m^2. Using M-20 grade concrete and Fe-415 HYSD bars, design one of the intermediate tee beam.

Future Study

https://nptel.ac.in/courses/105105105/11

Module – 4

Design of Staircases

Introduction

Objectives

Types of staircases

Structural classification

Problems

Outcomes

Assignment questions

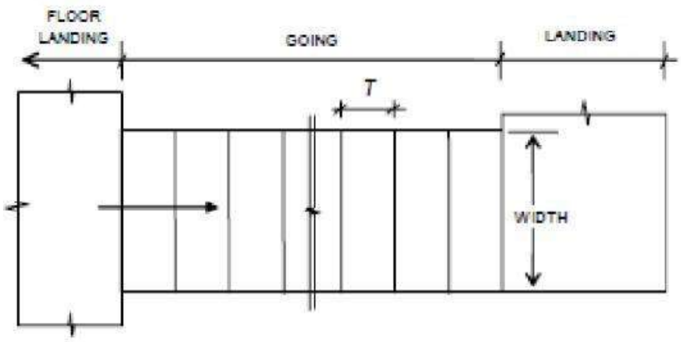
Future study

Introduction

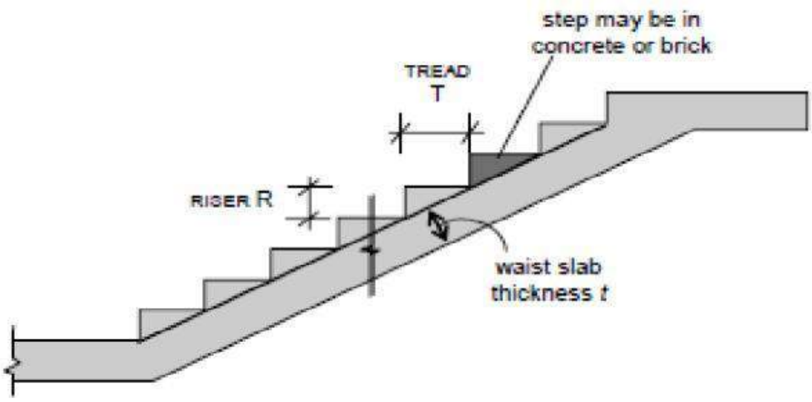
The staircase is an important component of a building, and often the only means of access between the various floors in the building. It consists of a *flight* of steps, usually with one or more intermediate *landings* (horizontal slab platforms) provided between the floor levels. The horizontal top portion of a step (where the foot rests) is termed *tread* and the vertical projection of the step (i.e., the vertical distance between two neighbouring steps) is called *riser*. Values of 300 mm and 150 mm are ideally assigned to the tread and riser respectively — particularly in public buildings. However, lower values of tread (up to 250 mm) combined with higher values of riser (up to 190 mm) are resorted to in residential and factory buildings. The *width* of the stair is generally around 1.1 – 1.6*m*, and in any case, should normally not be less than 850 mm; large stair widths are encountered in entrances to public buildings. The horizontal projection (plan) of an inclined flight of steps, between the first and last risers, is termed *going*. A typical flight of steps consists of two landings and one going, as depicted in Fig. Generally, risers in a flight should not exceed about 12 in number. The steps in the flight can be designed in a number of ways: with *waist slab*, with *tread-riser* arrangement (without waist slab) or with *isolated tread slabs* — as shown in Fig respectively.

Objectives

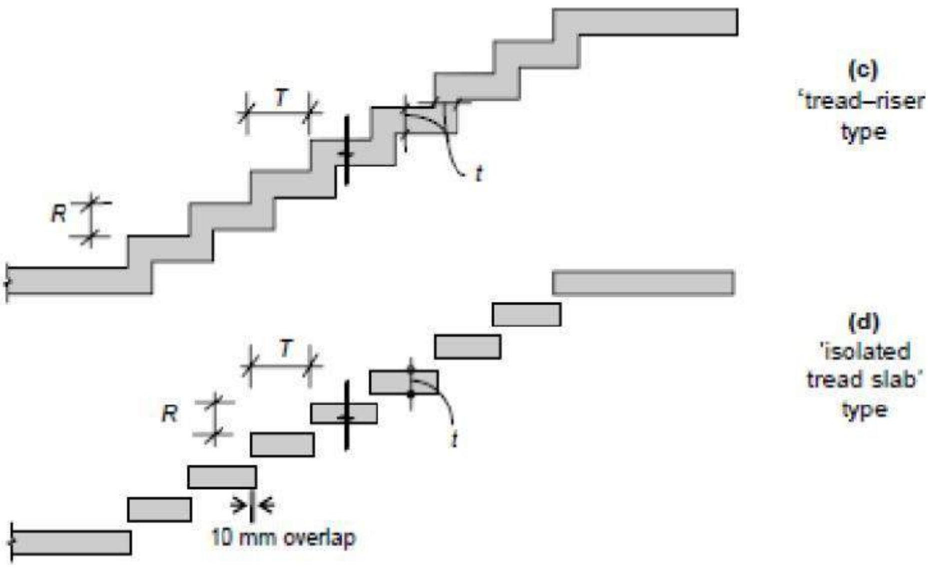
1. To design a dog-legged and open newel staircases



(a)
PLAN



(b)
'waist slab' type



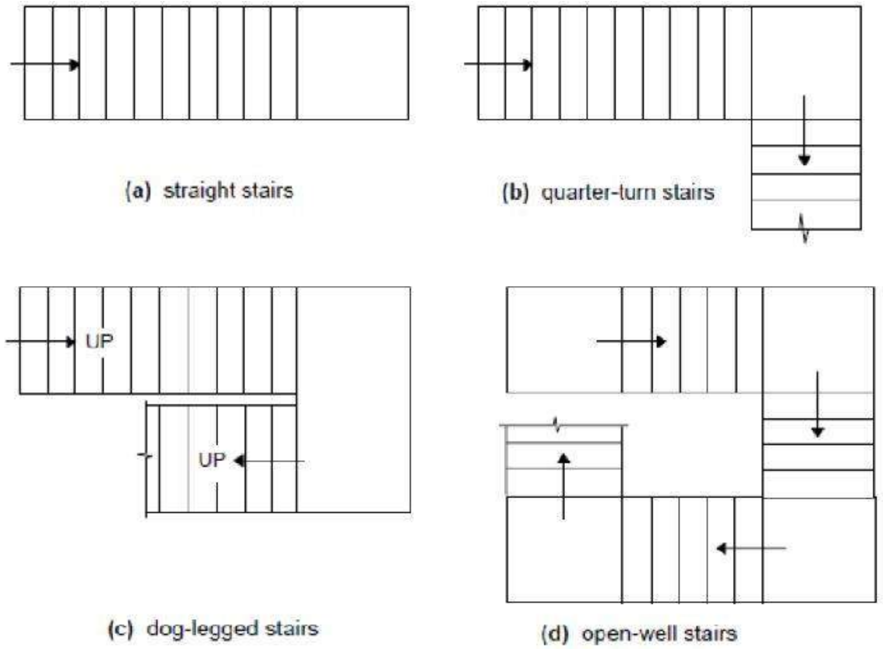
A typical flight in a staircase

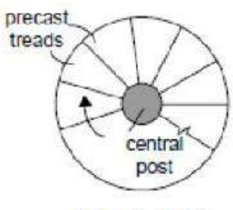
TYPES OF STAIRCASES

Geometrical Configurations

A wide variety of staircases are met with in practice. Some of the more common geometrical configurations are depicted in Fig. These include:

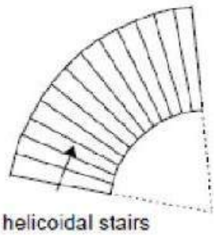
- Straight stairs (with or without intermediate landing)
- Quarter-turn stairs
- Dog-legged stairs
- Open well stairs
- Spiral stairs
- Helicoidal stairs





(e) spiral stairs

PLAN VIEWS



(f) helicoidal stairs

Structural Classification

Structurally, staircases may be classified largely into two categories, depending on the predominant direction in which the slab component of the stair undergoes flexure:

1. Stair slab spanning transversely (stair widthwise);
2. Stair slab spanning longitudinally (along the incline).

Stair Slab Spanning Transversely

The slab component of the stair (whether comprising an isolated tread slab, a tread-riser unit or a waist slab) is supported on its side(s) or cantilevers laterally from a central support. The slab supports gravity loads by bending essentially in a *transverse vertical plane*, with the span along the *width* of the stair.

In the case of the cantilevered slabs, it is economical to provide isolated treads (without risers). However, the tread-riser type of arrangement and the waist slab type are also sometimes employed in practice, as cantilevers. The spandrel beam is subjected to torsion (equilibrium torsion‘), in addition to flexure and shear.

When the slab is supported at the two sides by means of _stringer beams‘ or masonry walls, it may be designed as simply supported, but reinforcement at the top should be provided near the supports to resist the _negative‘ moments that may arise on account of possible partial fixity.

Stair Slab Spanning Longitudinally

In this case, the supports to the stair slab are provided parallel to the riser at two or more locations, causing the slab to bend longitudinally between the supports. It may be noted that longitudinal bending can occur in configurations other than the straight stair configuration, such as quarter-turn stairs, dog-legged stairs, open well stairs and helicoidal stairs.

The slab arrangement may either be the conventional waist slab type or the tread-riser type. The slab thickness depends on the _effective span‘, which should be taken as the centreto-centre distance between the beam/wall supports, according to the Code (Cl. 33.1a, c).In certain situations, beam or wall supports may not be available parallel to the riser at the landing. Instead, the flight is supported between the landings, which span transversely, parallel to the risers. In such cases, the Code(Cl. 33.1b) specifies that the effective span for the flight (spanning longitudinally) should be taken as the going of the stairs plus at each end either half the width of the landing or one metre, whichever is smaller.

Numerical Problem

Design a (waist slab type) dog-legged staircase for an office building, given the following data:

• Height between floor = 3.2 m;

• Riser = 160 mm, tread = 270 mm;

• Width of flight = landing width = 1.25 m

• Live load = 5.0 kN/m

• Finishes load = 0.6 kN/m

Assume the stairs to be supported on 230 mm thick masonry walls at the outer edges of the landing, parallel to the risers [Fig. 12.13(a)]. Use M 20 concrete and Fe 415 steel. Assume mild exposure conditions.

Solution

Given: R = 160 mm, T = 270 mm ⇒+RT22

= 314 mm Effective span = c/c distance between supports = 5.16 m [Fig below].

• Assume a waist slab thickness ≈l20 = 5160/20 = 258 →260 mm.

Assuming 20 mm clear cover (*mild* exposure) and 12 φ main bars,

effective depth d = 260 – 20 – 12/2 = 234 mm.

The slab thickness in the landing regions may be taken as 200 mm, as the bending moments are relatively low here.

Loads on going [fig. below] on projected plan area:

(1) self-weight of waist slab @ 25 × 0.26 × 314/270	= 7.56 kN/m^2
(2) self-weight of steps @ 25 × (0.5x0.16)	= 2.00 kN/m^2
(3) finishes (given)	= 0.60 kN/m^2
(4) live load (given)	= 5.00 kN/m^2
Total	=15.16 kN/m^2

⇒Factored load = 15.16 × 1.5 = 22.74 kN/m^2

• *Loads on landing*

(1) self-weight of slab @ 25 × 0.20 – 5.00 kN/m^2

(2) finishes @ 0.6 kN/m^2

(3) live loads @ 5.0 kN/m^2

Total =10.60 kN/m^2

⇒Factored load = 10.60 × 1.5 = 15.90 kN/m^2

• *Design Moment* [Fig. below]

Reaction R= (15.90x1.365)+(22.74x2.43)/2 = 49.33 kN/m

Maximum moment at midspan:

M_u = (49.33 × 2.58) – (15.90 × 1.365) × (2.58 – 1.365/2)

– (22.74) × (2.58 – 1.365)^2/2

= 69.30 kNm/m

$$R = \frac{M_u}{bd^2} = 1.265 \text{ MPa}$$

Assuming f_{ck} = 20 MPa, f_y = 415 MPa,

$$\frac{p_t}{100} = \frac{A_{st}}{100} = 0.381x10^{-2}$$

$$\Rightarrow (A_{st})_{req} = (0.381x10^{-2})x10^3x234 = 892\,m\!m$$

Required spacing of 12 φ bars = 127 mm

Required spacing of 16 φ bars = 225 mm

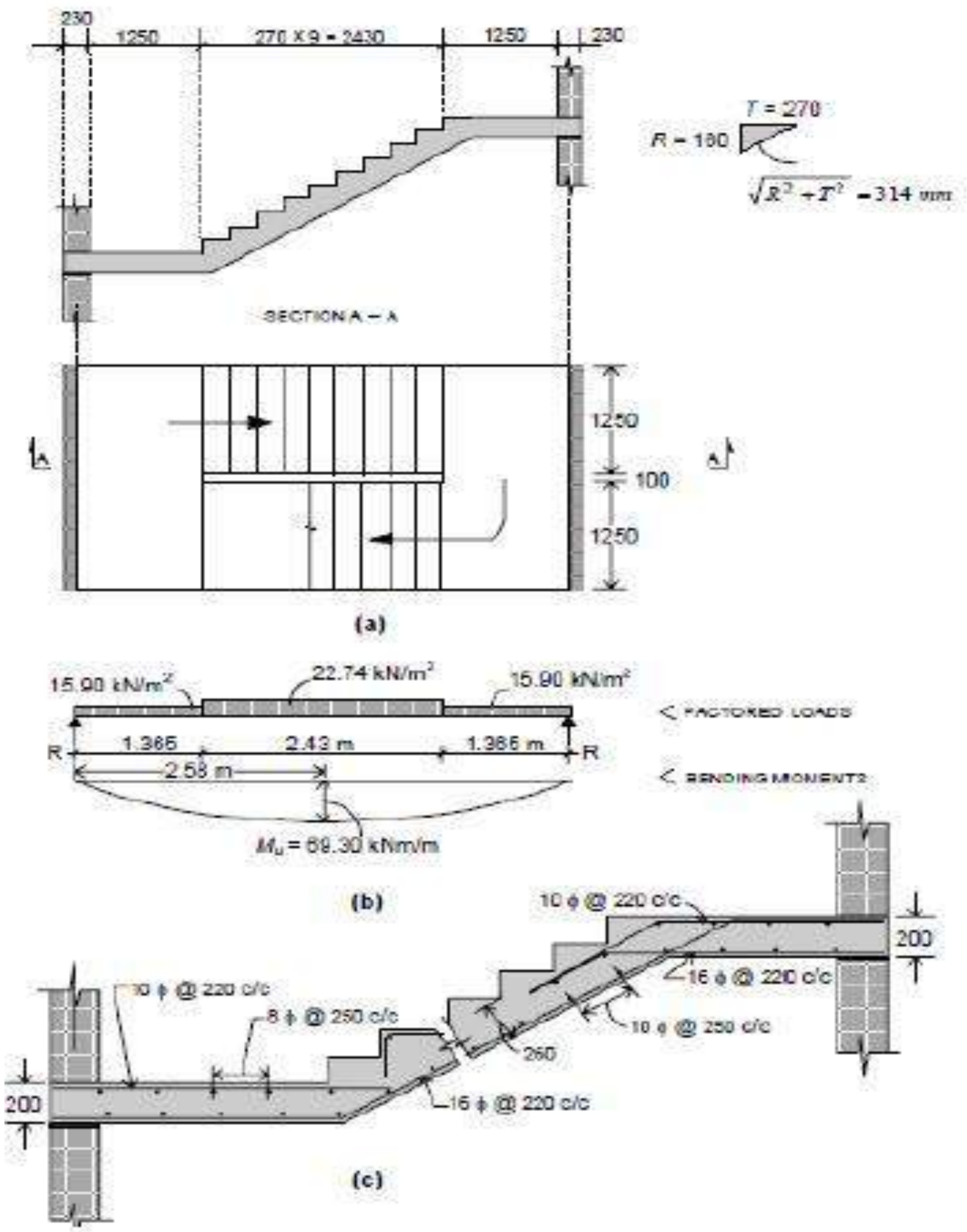
Provide 16 φ @ 220c/c

• *Distributors*

$$(A_{st})_{req} = 0.0012bt = 312\,mm^2\,/\,m$$

spacing 10 φ bars = 251 mm

Provide 10 φ @ 250c/c as distributors.



Outcome

1. Able to design the dog-legged and open newel staircase

Assignment Questions

Design a dog legged stair case for a residential building hall measuring 2.2m x 4.7 m. The width of the landing is 1m. The distance between floor to floor is 3.3 m. The rise and tread may be taken as 150mm and 270mm respectively. The weight of floor finish is 1 kN/m2. The materials used are M20 grade concrete and Fe415 grade steel. Sketch the details of steel. Here flight and the landing slabs spans in the same direction i.e, Flight spans longitudinally.

Future Study

https://nptel.ac.in/courses/105105104/pdf/m9l20.pdf

Slabs

Introduction

Objective

4.2.3Classification of Slabs

Method of analysis

General guidelines

Behavior of one-way slab

Behavior of two-way slab

Types of two-way slabs

Design example

Outcomes

Assignment question

Future study

Introduction to Slabs

A slab is a flat two dimensional planar structural element having thickness small compared to its other two dimensions. It provides a working flat surface or a covering shelter in buildings. It primarily transfer the load by bending in one or two directions. Reinforced concrete slabs are used in floors, roofs and walls of buildings and as the decks of bridges. The floor system of a structure can take many forms such as in situ solid slab, ribbed slab or pre-cast units. Slabs may be supported on monolithic concrete beam, steel beams, walls or directly over the columns. Concrete slab behave primarily as flexural members and the design is similar to that of beams.

Objective

1. To design one-way and two-way slabs

CLASSIFICATION OF SLABS

Slabs are classified based on many aspects

1) **Based of shape:** Square, rectangular, circular and polygonal in shape.

2) **Based on type of support:** Slab supported on walls, Slab supported on beams, Slab supported on columns (Flat slabs).

3) **Based on support or boundary condition:** Simply supported, Cantilever slab, Overhanging slab, Fixed or Continues slab.

4) **Based on use:** Roof slab, Floor slab, Foundation slab, Water tank slab.

5) **Basis of cross section or sectional configuration:** Ribbed slab /Grid slab, Solid slab, Filler slab, Folded plate

6) Basis of spanning directions:

One way slab – Spanning in one direction

Two way slab - Spanning in two direction

METHODS OF ANALYSIS

The analysis of slabs is extremely complicated because of the influence of number of factors stated above. Thus the exact (close form) solutions are not easily available. The various methods are:

a) Classical methods – Levy and Naviers solutions (Plate analysis)

b) Yield line analysis – Used for ultimate /limit analysis

c) Numerical techniques – Finite element and Finite difference method.

d) Semi empirical – Prescribed by codes for practical design which uses coefficients.

GENERAL GUIDELINES

a. Effective span of slab :

Effective span of slab shall be lesser of the two

1. l = clear span + d (effective depth)

2. l = Center to center distance between the support

b. Depth of slab:

The depth of slab depends on bending moment and deflection criterion. the trail depth can be obtained using:

- Effective depth d= Span /($(l/d)_{Basic}$ x modification factor)
- For obtaining modification factor, the percentage of steel for slab can be assumed from 0.2 to 0.5%
- The effective depth d of two way slabs can also be assumed using cl.24.1, IS 456

Type of support	Fe-250	Fe-415
Simply supported	l/35	l/28
continuous	l/40	l/32

OR

The following thumb rules can be used

- One way slab d= (l/22) to (l/28).
- Two way simply supported slab d= (l/20) to (l/30)
- Two way restrained slab d= (l/30) to (l/32)

c. Load on slab:

The load on slab comprises of Dead load, floor finish and live load. The loads are calculated per unit area (load/m^2).

Dead load = D x 25 kN/m^2 (Where D is thickness of slab in m)

Floor finish (Assumed as) = 1 to 2 kN/m^2

Live load (Assumed as) = 3 to 5 kN/m^2 (depending on the occupancy of the building)

DETAILING REQUIREMENTS AS PER IS 456: 2000

a. Nominal Cover:

For Mild exposure – 20 mm

For Moderate exposure – 30 mm

However, if the diameter of bar do not exceed 12 mm, or cover may be reduced by 5 mm.

Thus for main reinforcement up to 12 mm diameter bar and for mild exposure, the nominal cover is 15 mm

b. **Minimum reinforcement :** The reinforcement in either direction in slab shall not be less than

- 0.15% of the total cross sectional area for Fe-250 steel
- 0.12% of the total cross sectional area for Fe-415 & Fe-500 steel.

c. **Spacing of bars:** The maximum spacing of bars shall not exceed

- Main Steel – 3d or 300 mm whichever is smaller
- Distribution steel –5d or 450 mm whichever is smaller

Note: The minimum clear spacing of bars is not kept less than 75 mm (Preferably 100 mm) though code do not recommend any value.

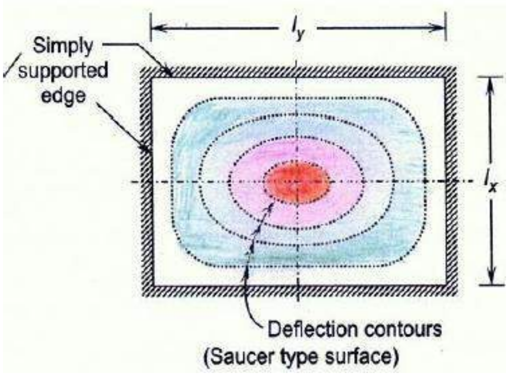
d. **Maximum diameter of bar**: The maximum diameter of bar in slab, shall not exceed D/8, where D is the total thickness of slab.

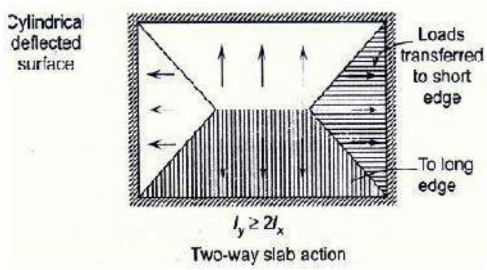
BEHAVIOR OF ONE WAY SLAB

When a slab is supported only on two parallel apposite edges, it spans only in the direction perpendicular to two supporting edges. Such a slab is called one way slab. Also, if the slab is supported on all four edges and the ratio of longer span(l_y) to shorter span (l_x) i.e ly/lx > 2, practically the slab spans across the shorter span. Such a slabs are also designed as one way slabs. In this case, the main reinforcement is provided along the spanning direction to resist one way bending.

BEHAVIOR OF TWO WAY SLABS

A rectangular slab supported on four edge supports, which bends in two orthogonal directions and deflects in the form of dish or a saucer is called two way slabs. For a two way slab the ratio of ly/lx shall be ≤ 2.0.





Since, the slab rest freely on all sides, due to transverse load the corners tend to curl up and lift up. The slab looses the contact over some region. This is known as lifting of corner. These slabs are called two way simply supported slabs. If the slabs are cast monolithic with the beams, the corners of the slab are restrained from lifting. These slabs are called restrained slabs. At corner, the rotation occurs in both the direction and causes the corners to lift. If the corners of slab are restrained from lifting, downward reaction results at corner & the end strips gets restrained against rotation. However, when the ends are restrained and the rotation of central strip still occurs and causing rotation at corner (slab is acting as unit) the end strip is subjected to torsion.

Types of Two Way Slab

Two way slabs are classified into two types based on the support conditions:

a) Simply supported slab

b) Restrained slabs

Two way simply supported slabs

The bending moments M_x and M_y for a rectangular slabs simply supported on all four edges with corners free to lift or the slabs do not having adequate provisions to prevent lifting of corners are obtained using

$M_x = \alpha_x W l_x$

$M_y = \alpha_y W l_x$

Where, α_x and α_y are coefficients given in Table 1 (Table 27,IS 456-2000)

W- Total load /unit area

l_x & l_y – lengths of shorter and longer span.

Two way restrained slabs

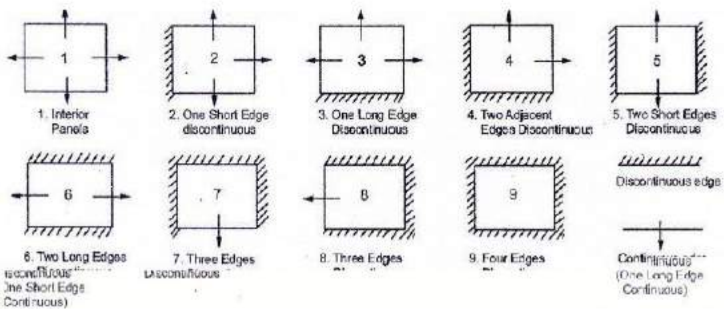
When the two way slabs are supported on beam or when the corners of the slabs are prevented from lifting the bending moment coefficients are obtained from Table 2 (Table 26, IS456-2000) depending on the type of panel shown in Fig. 3. These coefficients are obtained using yield line theory. Since, the slabs are restrained; negative moment arises near the supports. The bending moments are obtained using;

M_x (Negative) $\quad {}_x^{(-)} W l^2_x$

M_x (Positive) $\quad {}_x^{(+)} W l^2_x$

M_y (Negative) $\quad {}_y^{(-)} W l^2_x$

M_y (Positive) $\quad {}_y^{(+)} W l^2_x$



ONE WAY CONTINUOUS SLAB

The slabs spanning in one direction and continuous over supports are called one way continuous slabs. These are idealised as continuous beam of unit width. For slabs of uniform section which support substantially UDL over three or more spans which do not differ by more than 15% of the longest, the B.M and S.F are obtained using the coefficients available in Table 12 and Table 13 of IS 456-2000. For moments at supports where two unequal spans meet or in case where the slabs are not equally loaded, the average of the two values for the negative moments at supports may be taken. Alternatively, the moments may be obtained by moment distribution or any other methods.

DESIGN EXAMPLES

1. Design a simply supported one –way slab over a clear span of 3.5 m. It carries a live load of 4 kN/m2 and floor finish of 1.5 kN/m2. The width of supporting wall is 230 mm. Adopt M- 20 concrete & Fe-415 steel.

1) Trail depth and effective span

Assume approximate depth d =L/26

3500/26 = 134 mm

Assume overall depth D=160 mm & clear cover 15mm for mild exposure

d = 160-15 (cover) -10/2 (dia of Bar/2) =140 mm

Effective span is lesser of the two

i. l =3.5 + 0.23 (width of support) = 3.73 m

ii. l= 3.5 + 0.14 (effective depth) =3.64 m

effective span = 3.64 m

2) Load on slab

i. Self weight of slab = 0.16 x 25 = 4.00

ii. Floor finish = 1.50

iii. Live load = 4.00

= 9.5 kN/m^2

Ultimate load W_u = 9.5 x 1.5 = 14.25 kN/m^2

3) Design bending moment and check for depth

$$M_u = W_u l^2/8 = \frac{14.25X3.64^2}{8} = 23.60 \text{ kN/m}$$

Minimum depth required from BM consideration

$$d = \sqrt{\frac{Mu}{0.138 f_{ck} b}} = \sqrt{\frac{23.60X10^6}{0.138X20X1000}} = 92.4 > 140 \quad \text{(OK)}$$

4) Area of Reinforcement

Area of steel is obtained using the following equation

$$Mu = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

$$23.60X10^6 = 0.87X415XA_{st}X140\left(1 - \frac{415XA_{st}}{20X1000X140}\right)$$

$$23.60X10^6 = 50547A_{st} - 749A_{st}^2$$

Solving A_{st} =504mm^2

OR

$$A_{st} = \frac{0.5f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6M_u}{f_{ck}bd^2}}\right]bd$$

$$A_{st} = \frac{0.5X20}{415}\left[1 - \sqrt{1 - \frac{4.6X23.60X10^6}{20X1000X140^2}}\right]1000X140$$

$$= 505 \text{ mm}^2$$

Spacing of 10mm $S_V = \frac{a_{st}}{A_{st}} X1000$

$$S_V = \frac{78}{505} X1000 = 154 \text{ mm}$$

Provide 10mm @ 150 C/C ($< 3d$ or 300)

(420 or 300) OK

Provided steel (A_{st}=524mm^2,P_t=0.37%)

Distribution steel@ 0.12% of the Gross area.

$\frac{0.12}{100} X1000X160$ =192 mm^2

Spacing of 8 mm $S_V=\frac{50}{192} X1000$=260 mm

Provide 8 mm @260 mm C/C (<5d or 450)

(700 or 450) OK

5) Check for shear

Design shear $V_u=W_u l/2$

$=14.25X\frac{3.64}{2}$= 25.93 kN

$$\tau_v = \frac{25.93X10^3}{1000X140} = 0.18\ N/mm^2 \qquad (<\tau_{c\ max} = 2.8\ N/mm^2)$$

Shear resisted by concrete $\tau_c = 0.42\ for\ p_t = 0.37$ (Table 19, IS 456-2000)

However for solid slab design shear strength shall be

$= \tau_c k$

Where, K is obtained from Cl.40.2.1.1, IS 456 -2000

$\tau_{cd} = 0.42X1.28 = 0.53\ N/mm^2$

$\tau_{cd} > \tau_v$ OK

6) Check for deflection

$$\left(\frac{l}{d}\right)_{Actual} < \left(\frac{l}{d}\right)_{Allowable}$$

$$\left(\frac{l}{d}\right)_{Allowable} = \left(\frac{l}{d}\right)_{Basic} Xk_1Xk_2Xk_3Xk_4.$$

k_1- Modification factor for tension steel

k_2 Modification factor for compression steel

k_3 Modification factor for T-sections k_4-Only if span exceeds 10 m (10/span)

$k_1 = 1.38\ for\ P_t = 0.37$ (Fig. 4,cl.32.2.1)

$$\left(\frac{l}{d}\right)_{Allowable} = 20X1.38 = 27.6$$

$$\left(\frac{l}{d}\right)_{Actual} = 3630/140 = 25.92$$

$$\left(\frac{l}{d}\right)_{Actual} < \left(\frac{l}{d}\right)_{Allowable} \qquad \text{(OK)}$$

7) **Check for Development length**

Development length

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

L_d = (0.87x415x10) / (4x1.2x1.6) =470 mm

At simple support, where compressive reaction confines the bars, to limit the dia. of bar

$$L_d \leq 1.3 \left(\frac{M_1}{V}\right) + L_o$$

Since alternate bars are cranked $M_1=M_u/2$ = 23.2/2 = 11.8 kN.m

V_1 = 5.93 kN., Providing 90o bend and 25 mm end cover

L_o = 230/2 25 + 3(dia of bar) = 120

470 < (1.3x11.8x106) / (25.9x103) + 120 = 711 mm O. K.

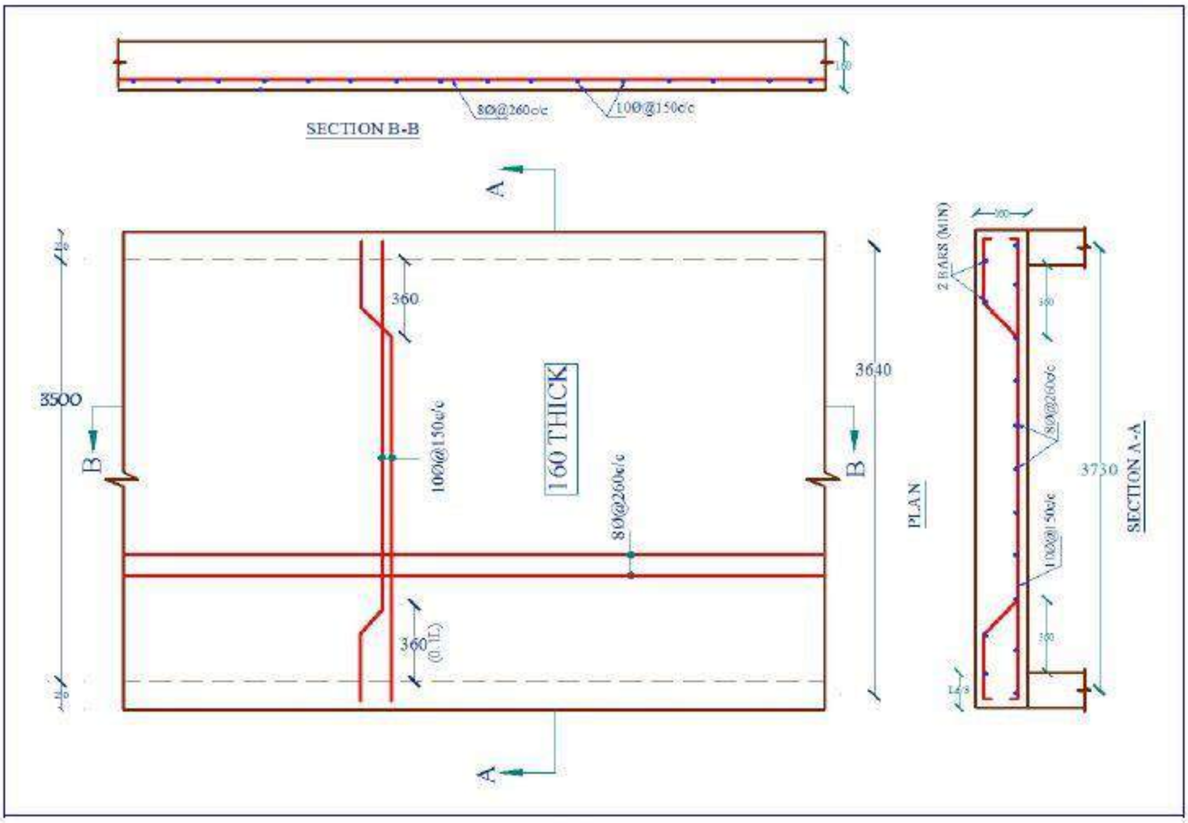
However, from the end anchorage requirement

extend the bars for a length equal to ld/3 = 156 mm from inner face of support

8) **Check for cracking**

- Steel is more than 0.12% of the gross area.
- Spacing of steel is < 3d
- Diameter of bar used is < 160/8=20mm

Check for cracking is satisfied.



Reinforcement Detail of One way slab

Outcome

1. Able to design one-way and two-way slab

Assignment questions

1. Design a R.C Slab for a room measuring 6.5mX5m. The slab is cast monolithically over the beams with corners held down. The width of the supporting beam is 230 mm. The slab carries superimposed load of 4.5kN/m2. Use M-20 concrete and Fe-500 Steel.

Future Study

https://nptel.ac.in/courses/105105104/pdf/m8l19.pdf

Module – 5
Design of Columns

Introduction
Objective
Definitions
Classification of columns
Longitudinal reinforcement
Transverse reinforcement
Problems
Outcome
Assignment questions
Future study

Introduction

Compression members are structural elements primarily subjected to axial compressive forces and hence, their design is guided by considerations of strength and buckling. Examples of compression member pedestal, column, wall and strut.

Objective

To design the RCC rectangular and circular columns as per the codal provisions

Definitions

(a) Effective length: The vertical distance between the points of inflection of the compression member in the buckled configuration in a plane is termed as effective length l_e of that compression member in that plane. The effective length is different from the unsupported length l of the member, though it depends on the unsupported length and the type of end restraints. The relation between the effective and unsupported lengths of any compression member is $l_e = k\,l$ (1) Where k is the ratio of effective to the unsupported lengths. Clause 25.2 of IS 456 stipulates the effective lengths of compression members (vide Annex E of IS 456). This parameter is needed in classifying and designing the compression members.

(b) Pedestal: Pedestal is a vertical compression member whose effective length l_e does not exceed three times of its least horizontal dimension b (cl. 26.5.3.1h, Note). The other horizontal dimension D shall not exceed four times of b.

(c) Column: Column is a vertical compression member whose unsupported length l shall not exceed sixty times of b (least lateral dimension), if restrained at the two ends. Further, its unsupported length of a cantilever column shall not exceed $100b/D$, where D is the larger lateral dimension which is also restricted up to four times of b (vide cl. 25.3 of IS 456).

(d) Wall: Wall is a vertical compression member whose effective height H_{we} to thickness t (least lateral dimension) shall not exceed 30 (cl. 32.2.3 of IS 456). The larger horizontal dimension i.e., the length of the wall L is more than 4t.

Classification of Columns Based on Types of Reinforcement

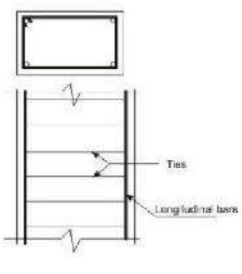


Figure 3.1(a) Tied Column

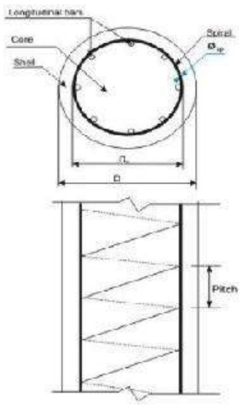


Figure 3.1(b) Column with helical reinforcement

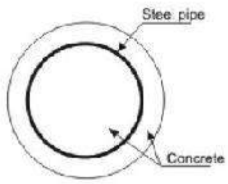


Figure 3.1(d) Composite column (steel pipe)

Figure 3.1 Tied, helically bound and composite columns

Based on the types of reinforcement, the reinforced concrete columns are classified into three groups:

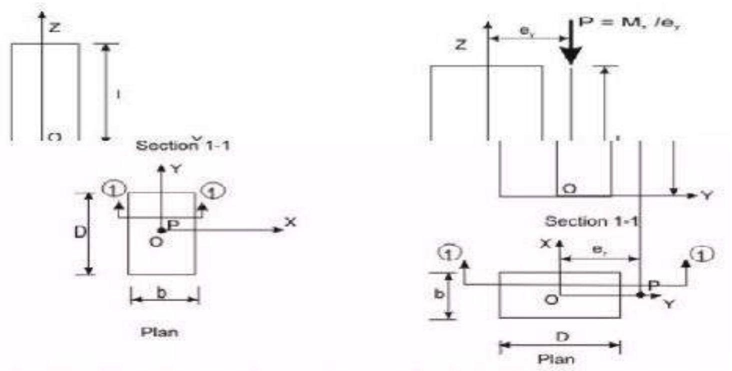
(i) Tied columns: The main longitudinal reinforcement bars are enclosed within closely spaced lateral ties (Fig.3.1a).

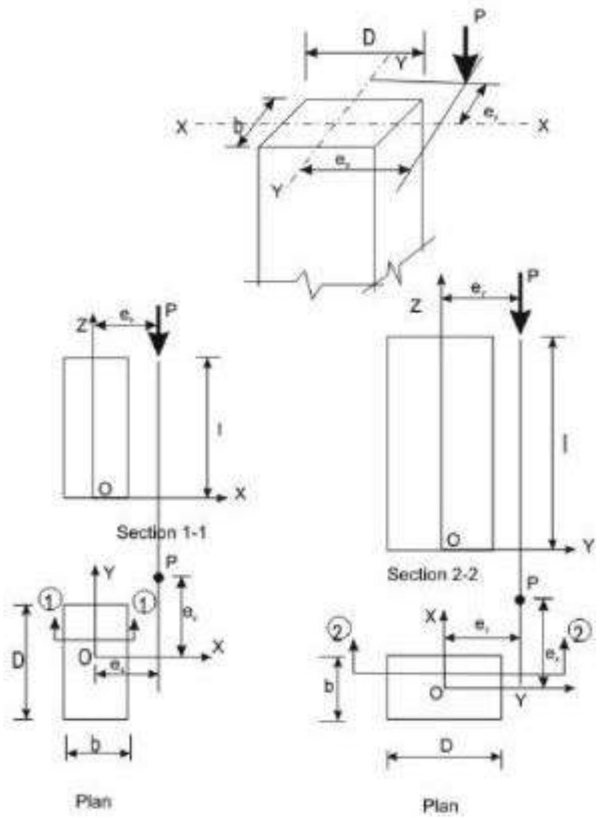
(ii) Columns with helical reinforcement: The main longitudinal reinforcement bars are enclosed within closely spaced and continuously wound spiral reinforcement. Circular and octagonal columns are mostly of this type (Fig. 3.1b).

(iii) Composite columns: The main longitudinal reinforcement of the composite columns consists of structural steel sections or pipes with or without longitudinal bars (Fig. 3.1c and d).

Out of the three types of columns, the tied columns are mostly common with different shapes of the cross-sections viz. square, rectangular etc. Helically bound columns are also used for circular or octagonal shapes of cross-sections.

Based on Loadings





Columns are classified into the three following types based on the loadings:
(i) Columns subjected to axial loads only (concentric), as shown in Fig. 3.2a.
(ii) Columns subjected to combined axial load and uniaxial bending, as shown in Fig. 3.2b.
(iii) Columns subjected to combined axial load and bi-axial bending, as shown in Fig. 3.2c.

Based on Slenderness Ratios

Columns are classified into the following two types based on the slenderness ratios:

(i) Short columns

(ii) Slender or long columns

The slenderness ratio of steel column is the ratio of its effective length l_e to its least radius of gyration *r*. In case of reinforced concrete column, however, IS 456 stipulates the slenderness ratio as the ratio of its effective length l_e to its least lateral dimension. As mentioned earlier in sec. 3.1(a), the effective length l_e is different from the unsupported length, the rectangular reinforced concrete column of cross-sectional dimensions *b* and *D* shall have two effective lengths in the two directions of *b* and *D*. Accordingly, the column may have the possibility of buckling depending on the two values of slenderness ratios as given below:

Slenderness ratio about the major axis = l_{ex}/D

Slenderness ratio about the minor axis = l_{ey}/b

Based on the discussion above, cl. 25.1.2 of IS 456 stipulates the following:

A compression member may be considered as short when both the slenderness ratios l_{ex}/D and l_{ey}/b are less than 12 where l_{ex} = effective length in respect of the major axis, D = depth in respect of the major axis, l_{ey}= effective length in respect of the minor axis, and b = width of the member. It shall otherwise be considered as a slender compression member.

Further, it is essential to avoid the mode 3 type of failure of columns so that all columns should have material failure (modes 1 and 2) only. Accordingly, cl. 25.3.1 of IS 456 stipulates the maximum unsupported length between two restraints of a column to sixty times its least lateral dimension. For cantilever columns, when one end of the column is unrestrained, the unsupported length is restricted to $100b/D$ where b and D are as defined earlier.

Longitudinal Reinforcement

The longitudinal reinforcing bars carry the compressive loads along with the concrete. Clause 26.5.3.1 stipulates the guidelines regarding the minimum and maximum amount, number of bars, minimum diameter of bars, spacing of bars etc. The following are the salient points:

(a) The minimum amount of steel should be at least 0.8 per cent of the gross cross-sectional area of the column required if for any reason the provided area is more than the required area.

(b) The maximum amount of steel should be 4 per cent of the gross cross-sectional area of the column so that it does not exceed 6 per cent when bars from column below have to be lapped with those in the column under consideration.

(c) Four and six are the minimum number of longitudinal bars in rectangular and circular columns, respectively.

(d) The diameter of the longitudinal bars should be at least 12 mm.

(e) Columns having helical reinforcement shall have at least six longitudinal bars within and in contact with the helical reinforcement. The bars shall be placed equidistant around its inner circumference.

(f) The bars shall be spaced not exceeding 300 mm along the periphery of the column.

(g) The amount of reinforcement for pedestal shall be at least 0.15 per cent of the cross sectional area provided.

Transverse Reinforcement

Transverse reinforcing bars are provided in forms of circular rings, polygonal links (lateral ties) with internal angles not exceeding 135^0 or helical reinforcement. The transverse reinforcing bars are provided to ensure that every longitudinal bar nearest to the compression face has effective lateral support against buckling. Clause 26.5.3.2 stipulates the guidelines of the arrangement of transverse reinforcement. The salient points are:

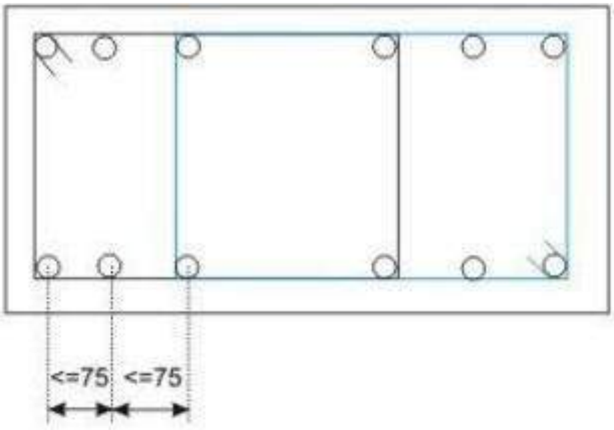


Figure 3.4 Lateral tie (Arrangement 1)

Pitch and Diameter of Lateral Ties

(a) Pitch: The maximum pitch of transverse reinforcement shall be the least of the following:

(i) The least lateral dimension of the compression members;

(ii) Sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and

(iii) 300 mm.

(b) Diameter: The diameter of the polygonal links or lateral ties shall be not less than one fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.

Assumptions in the Design of Compression Members by Limit State of Collapse The following are the assumptions in addition to given in 38.1 (a) to (e) for flexure for the design of compression members (cl. 39.1 of IS 456).

(i) The maximum compressive strain in concrete in axial compression is taken as 0.002.

(ii) The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

Minimum Eccentricity

In practical construction, columns are rarely truly concentric. Even a theoretical column loaded axially will have accidental eccentricity due to inaccuracy in construction or variation of materials etc. Accordingly, all axially loaded columns should be designed considering the minimum eccentricity as stipulated in cl. 25.4 of IS 456 and given below (Fig.3.2c)

$e_{x\,min} \geq$ greater of ($l/500 + D/30$) or 20 mm

$e_{y\,min} \geq$ greater of ($l/500 + b/30$) or 20 mm where l, D and b are the unsupported length, larger lateral dimension and least lateral dimension, respectively.

Governing Equation for Short Axially Loaded Tied Columns

Factored concentric load applied on short tied columns is resisted by concrete of area A_c and longitudinal steel of areas A_{sc} effectively held by lateral ties at intervals. Assuming the design strengths of concrete and steel are $0.4f_{ck}$ and $0.67f_y$, respectively, we can write

$P_u = 0.4f_{ck}A_c + 0.67f_yA_{sc}$

Where P_u = factored axial load on the member,

f_{ck} = characteristic compressive strength of the concrete,

A_c= area of concrete,

f_y = characteristic strength of the compression reinforcement, and

A_{sc}= area of longitudinal reinforcement for columns.

The above equation, given in cl. 39.3 of IS 456, has two unknowns Ac and A_{sc} to be determined from one equation. The equation is recast in terms of A_g, the gross area of concrete and p, the percentage of compression reinforcement employing

$A_{sc}= pA_g/100$

$A_c= A_g(1 – p/100)$

Accordingly, we can write

$P_u/A_g= 0.4f_{ck}+ (p/100) (0.67f_y– 0.4f_{ck}$

Equation 4 can be used for direct computation of A_g when P_u, f_{ck} and f_y are known by assuming p ranging from 0.8 to 4 as the minimum and maximum percentages of longitudinal reinforcement. Equation 10.4 also can be employed to determine A_g and p in a similar manner by assuming p.

Numerical Problem

Design the reinforcement in a column of size 400 mm x 600 mm subjected to an axial load of 2000 kN under service dead load and live load. The column has an unsupported length of 4.0 m and effectively held in position and restrained against rotation in both ends. Use M 25 concrete and Fe 415 steel.

Solution

Step 1: To check if the column is short or slender

Given l = 4000 mm, b = 400 mm and D = 600 mm. Table 28 of IS 456 = $l_{ex} = l_{ey} = 0.65(l)$ = 2600 mm. So, we have

$l_{ex}/D = 2600/600 = 4.33 < 12$

$l_{ey}/b = 2600/400 = 6.5 < 12$

Hence, it is a short column.

Step 2: Minimum eccentricity

$e_{x\,min}$ = Greater of ($l_{ex}/500 + D/30$) and 20 mm = 25.2 mm

$e_{y\,min}$ = Greater of ($l_{ey}/500 + b/30$) and 20 mm = 20 mm

$0.05\,D = 0.05(600) = 30$ mm > 25.2 mm (= $e_{x\,min}$)

$0.05\,b = 0.05(400) = 20$ mm = 20 mm (= $e_{y\,min}$)

Hence, the equation given in cl.39.3 of IS 456 (Eq.(1)) is applicable for the design here.

Step 3: Area of steel

Fro Eq.10.4, we have

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$3000(10^3) = 0.4(25)\{(400)(600) - A_{sc}\} + 0.67(415) A_{sc}$$

which gives,

$$A_{sc} = 2238.39 \text{ mm}^2$$

Provide 6-20 mm diameter and 2-16 mm diameter rods giving 2287 mm^2 (> 2238.39 mm^2) and p = 0.953 per cent, which is more than minimum percentage of 0.8 and less than maximum percentage of 4.0. Hence, o.k.

Step 4: Lateral ties

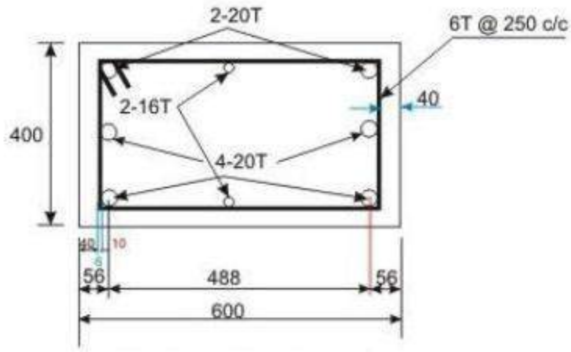
The diameter of transverse reinforcement (lateral ties) is determined from cl.26.5.3.2 C-2 of IS 456 as not less than (i) θ/4 and (ii) 6 mm. Here, θ = largest bar diameter used as longitudinal reinforcement = 20 mm. So, the diameter of bars used as lateral ties = 6 mm.

The pitch of lateral ties, as per cl.26.5.3.2 C-1 of IS 456, should be not more than the least of

(i) The least lateral dimension of the column = 400 mm

(ii) Sixteen times the smallest diameter of longitudinal reinforcement bar to be tied = 16(16) = 256 mm

(iii) 300 mm



Reinforcement Detailing

IS 456 recommends the following simplified method, based on Bresler's formulation, for the design of biaxially loaded columns. The relationship between M_{uxz} and M_{uyz} for a particular value of $P_u = P_{uz}$, expressed in non-dimensional form is:

$$(M_{ux} / M_{ux1})^{\alpha^n} + (M_{uy} / M_{uy1})^{\alpha^n} \leq 1 \qquad (5)$$

where M_{ux} and M_{uy} = moments about x and y axes due to design loads, and

α^n is related to P_u/P_{uz},

where

$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$

$$= 0.45\, A_g + (0.75 f_y - 0.45 f_{ck})\, A_{sc} \qquad (6)$$

where A_g = gross area of the section, and

A_{sc} = total area of steel in the section

M_{uxz}, M_{uyz}, M_{ux1} and M_{uy1} are explained earlier.

$\alpha^n = 1.0$, when $P_u/P_{uz} \leq 0.2$

$$\alpha^n = 0.67 + 1.67\, P_u/P_{uz}, \text{ when } 0.2 < (P_u/P_{uz}) < 0.8 \qquad (7)$$

$\alpha^n = 2.0$, when $(P/P\ ) \geq 0.8$

Outcome

1. Able to design the RCC columns

Assignment questions

Design the reinforcement to be provided in the short column is subjected to P_u= 2000 kN,

M_{ux}= 130 kNm (about the major principal axis) and M_{uy}= 120 kNm (about the minor principal axis). The unsupported length of the column is 3.2 m, width b = 400 mm and depth D = 500 mm. Use M 25 and Fe 415 for the design.

Future Study

https://nptel.ac.in/courses/105105105/